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AN IMPROVED DETERMINISTIC METHOD
FOR
FINDING RELATIVE ORBITAL STATE
FROM ANGLE MEASUREMENTS

By

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ABSTRACT

A deterministic method is presented for finding the relative state between two orbiting bodies. Three or four sextant measurements of angle data, plus a thrust maneuver some time between the first and last measurement, are the necessary inputs. Range measurements are not required. Use of a curvilinear coordinate system improves the accuracy relative to a Cartesian coordinate system which otherwise is useful for the same task. The resulting high-quality state-determination can be used as the "a priori estimate" in a stochastic, recursive state-estimator which uses angle-only data.

INTRODUCTION

The object of the analysis and computational algorithm described herein is to make an accurate, deterministic estimate of the relative state of two spacecraft orbiting a central body, such as the earth or moon, using only angle-data, when the component of separation range along the reference orbit is a few hundred miles or less. A succession of state-determinations by this method could be used to carry out rendezvous. Alternatively, an estimate made by this method could be used to initialize a stochastic state-estimator (such as a Kalman filter) which combines redundant angle-measurements in an optimal way to refine an a priori estimate. The highly accurate "a priori estimate" which the present method produces can improve the accuracy and convergence speed of the stochastic estimator.

Previous studies have shown that it is possible to determine relative orbital state from a sequence of measurements of the angle as seen at one vehicle between the line of sight to the other vehicle and identified stars, provided that one of the vehicles makes a thrust maneuver of known magnitude and direction at some time during the sequence^(1, 2). Reference 3 demonstrates that these methods are sufficiently accurate to carry out rendezvous from separation ranges as high as 150 nautical miles (nm). However, model inaccuracies begin to creep in at ranges beyond 50 nm, and it requires extra fuel and/or time because of them.

The dynamical model which is used in these references is a linearized expansion around a reference orbit. Defining the maneuvering vehicle as the interceptor and the non-maneuvering vehicle as the target is described by three Cartesian coordinates, x, y, z , in a target-centered coordinate system.* One of the sources of error in this dynamical model, and the principal cause of inaccuracy at ranges about 50 nm, is "curvature error." Curvature error is defined as the error in state which results from using Cartesian rather than curvilinear coordinates as the basis for the dynamical model.

For this paper, attention is restricted to circular reference orbits. If the target orbit has a small, but non-zero eccentricity, then the reference orbit is that coplanar circular orbit whose period equals that of the target orbit. By these rules, the resulting analysis, although carried out for the circular case can be applied directly to more general situations.

A curvilinear coordinate system, with axes u, v, w , centered at the target, is compared with the Cartesian coordinate system, x, y, z , in Fig. 1. In this figure, which shows only the in-plane geometry, the x -axis is forward and horizontal at the target position; y is perpendicular to the reference orbit plane, and z is the upward vertical through the target. Thus x measures the

* Free use will be made of the concepts, nomenclature, and symbols of Ref. 1, with which it is assumed that the reader is familiar.

downrange component of range, y measures the interceptor out-of-plane distance, and z measures the interceptor (relative) altitude. The curvilinear system also has its origin O at the target. The u -axis is curved to follow the reference orbit in the forward direction. The v -axis is normal to the plane of the reference orbit, and is therefore identical to y . The w -axis is radial. The u, v, w coordinates of the interceptor, whose projection into the reference plane defines the point b , are given by the linear length of the arc Od , the y -distance, and the distance db respectively, and are referred to as the "downrange distance," "out-of-plane distance," and (relative) "altitude." The interceptor position defines the central angle ϕ , the slant in-plane range ρ , and the line-of-sight angles δ and σ . All quantities are positive as illustrated.

It is shown⁽⁴⁾ that the state vector given in terms of the u, v, w position and position-derivatives propagates according to the self-same law as the state vector in Cartesian coordinates, namely

$$\begin{bmatrix} u & v & w & u' & v' & w' \end{bmatrix}_{\tau_i}^T = \Phi_{ij} \begin{bmatrix} u & v & w & u' & v' & w' \end{bmatrix}_{\tau_j}^T \quad (1)$$

where Φ_{ij} is the state transition matrix defined in Ref. 1 evaluated for $\tau_{ij} \equiv \tau_i - \tau_j$.

A simple example illustrates the value of the curvilinear system. Suppose the target and the spacecraft were in the

same circular orbit, and separated by 90° of earth-central angle. In the Cartesian system, there would be a sizable z -component (equal to R) suggesting an altitude differential which does not in fact exist. This is one form of the curvature error referred to earlier. In the curvilinear system, the downrange distance, u , would be $\pi R/2$, and the relative altitude, w , would be zero, as they should be.

The disadvantage of the curvilinear system, and the reason it was not adopted as the basic coordinate system in the rendezvous method of Ref. 1, is that the ratio of coordinates for points along a given line emanating from the origin is not invariant with range, as it is for the Cartesian coordinate system. For example, the ratio of the z to x coordinates of the point b in Fig. 1 is the same as the ratio of the z to x coordinates of point g , along the same line-of-sight. Both are equal to $\tan \delta$, the in-plane angle. But the ratio of w to u for point b is not equal to the ratio of w to u for point g . Without this invariance, the non-dimensionalizing procedure of Ref. 1 and the resulting simplified calculations for determining range from the calibrated thrust maneuver are not directly applicable to the curvilinear system.

The state vector $[u, v, w, u', v', w']^T$ can be propagated using the linear dynamics of equation (1) with high accuracy for any value of u , as long as v and w are much smaller than the reference orbit radius, even over a quarter- or half-orbit of downrange distance. The state vector in terms of Cartesian coordinates

$[x, y, z, x', y', z']^T$ can be propagated with high accuracy only when all three components of position are small, i. e. in a small spherical volume surrounding the target. The difference in accuracy between these two representations of state is another manifestation of the curvature error. Because of the greater accuracy of the curvilinear coordinates for large separations between target and interceptor, it is desirable to use these coordinates in solving the state-determination problem.

APPROACH

The method for determining state in curvilinear coordinates will now be described qualitatively. The state-determination problem is first solved in Cartesian coordinates, using any of the alternative methods of Refs. 1 and 2, based on a series of δ, σ measurements and the calibrated thrust maneuver. One is forced to Cartesian coordinates initially because the range at each measurement time is completely unknown. Although the solution so obtained has more or less curvature error in it, depending on the downrange distance, it nevertheless provides an estimate of the state and hence the range at each measurement. This range can be used to convert the measurements to what they would have been had it been possible to measure the range-dependent "line-of-sight curvi-angles" $\alpha \equiv \tan^{-1}(w/u)$ and $\nu \equiv \tan^{-1} v / \sqrt{u^2 + w^2}$, rather than the actual line-of-sight angle $\delta \equiv \tan^{-1}(z/x)$ and $\sigma \equiv \tan^{-1} y / \sqrt{x^2 + z^2}$. Then state

is redetermined using the corrected measurements in the same computational algorithm as before. The same algorithm applies because u, v, w propagate according to the same equation as x, y, z . However, the answer to the second state-determination is a result in curvilinear coordinates, and it will be more accurate than the first because the state-propagation equation (1) has no curvature error associated with it. Again, one obtains from this solution an estimate of the range at each measurement point, and since it is more accurate than the first estimate, a repetition of the prior conversion of line-of-sight angles to curvilinear angles will give still better results. By this means, an iterative process is established which converges to the true state in the curvilinear coordinates, to the accuracy of the linear model.

A plausibility argument can be advanced for the convergence of this bootstrap approach. One can solve for state in the original Cartesian coordinates without range data as an input, in which an estimate of range is obtained as an output. This estimate is ordinarily reasonable, if not exact, possessing only the curvature error, which may be just a few percent. Since an estimate of range was produced without knowing range at all it should be possible to produce a better estimate if range is known beforehand to some moderate level of accuracy, and so on through iterations to a convergence.

GEOMETRICAL RELATIONS

The equations relating Cartesian to curvilinear coordinates, and vice versa, are developed in this section. Referring to Fig. 1 it is seen that

$$\tan \varphi = \frac{ab}{QO + Oa} \quad (2)$$

or

$$\varphi = \tan^{-1} \frac{x}{R + z} \quad (-\pi < \varphi \leq \pi) \quad (3)$$

Also, from the geometry of the figure,

$$\varphi = \frac{u}{R} \quad (4)$$

Equating the last two results leads to the following,

$$u = R \tan^{-1} \left(\frac{x}{R + z} \right) \quad (5)$$

where R , the radius of the reference orbit, is a known positive constant.

Using the Pythagorean theorem for triangle Qba,

$$(R + w)^2 = x^2 + (R + z)^2 \quad (6)$$

Solving for w yields

$$w = \sqrt{x^2 + (R + z)^2} - R \quad (7)$$

Equations (5) and (7) give the curvilinear in terms of the Cartesian coordinates.

Solving (3) and (4) for x ,

$$x = (R + z) \tan \frac{u}{R} \quad (8)$$

Also, from triangle Qab,

$$(R + w) \cos \frac{u}{R} = (R + z) \quad (9)$$

from which

$$z = (R + w) \cos \frac{u}{R} - R \quad (10)$$

Substituting (10) into (8) and simplifying yields

$$x = (R + w) \sin \frac{u}{R} \quad (11)$$

Equations (10) and (11) give the Cartesian in terms of the curvilinear coordinates.

The definitions of the in-plane range ρ and in-plane line-of-sight angle δ of the interceptor relative to the x -axis are defined by the equations:

$$\rho \equiv \sqrt{x^2 + z^2} \quad (\rho \geq 0) \quad (12)$$

$$\tan \delta \equiv \frac{z}{x} \quad (-\pi < \delta \leq \pi) \quad (13)$$

It is convenient to define a set of variables which play the same role in the curvilinear system as these variables do in the Cartesian

system. To this end, define the in-plane curvi-range λ and in-plane line-of-sight curvi-angle α as follows:

$$\lambda \equiv \sqrt{u^2 + w^2} \quad (\lambda \geq 0) \quad (14)$$

$$\tan \alpha \equiv \frac{w}{u} \quad (-\pi < \alpha \leq \pi) \quad (15)$$

Then it follows that

$$u = \lambda \cos \alpha \quad (16)$$

$$w = \lambda \sin \alpha \quad (17)$$

Of course the line-of-sight angle δ is measurable directly (assuming the presence of a star on the local horizontal of the point O, Fig. 1) whereas the line-of-sight curvi-angle α and its tangent are only obtainable through computation. The next step is to derive α and λ as functions of δ and ρ , and vice versa. Substituting (5) and (7) into (15) yields,

$$\tan \alpha \equiv \frac{\sqrt{x^2 + (R+z)^2} - R}{R \tan^{-1} \left(\frac{x}{R+z} \right)} \quad (18)$$

Using

$$x = \rho \cos \delta \quad (19)$$

$$z = \rho \sin \delta \quad (20)$$

to eliminate x and z in (18), and defining the ratio ρ/R as β

$$\beta \equiv \frac{\rho}{R} \quad (\beta \geq 0) \quad (21)$$

the last result reduces to

$$\tan \alpha = \frac{\sqrt{\beta^2 \cos^2 \delta + (1 + \beta \sin \delta)^2} - 1}{\tan^{-1} \left(\frac{\beta \cos \delta}{1 + \beta \sin \delta} \right)} \quad (22)$$

This is a non-dimensionalized form of the sought relationship for $\alpha = \alpha(\rho, \delta)$. The angle φ can be obtained from (3), (19), (20), and (21) as follows:

$$\varphi = \tan^{-1} \left(\frac{\beta \cos \delta}{1 + \beta \sin \delta} \right) \quad (23)$$

Define a nondimensional form for the in-plane curvi-range

$$\mu = \frac{\lambda}{R} \quad (\mu \geq 0) \quad (24)$$

Then φ can be written

$$\varphi = \frac{\mu}{R} = \frac{\lambda \cos \alpha}{R} = \mu \cos \alpha \quad (25)$$

Now λ in (14) can be non-dimensionalized as follows:

$$\frac{\lambda}{R} = \mu = \sqrt{\left(\frac{u}{R} \right)^2 + \left(\frac{w}{R} \right)^2} \quad (26)$$

Using (5) and (7) in (26), replacing x and z with (19) and (20) and eliminating R with (21) yields

$$\mu = \left\{ \left[\tan^{-1} \left(\frac{\beta \cos \delta}{1 + \beta \sin \delta} \right) \right]^2 + \left[\sqrt{(\beta \cos \delta)^2 + (1 + \beta \sin \delta)^2} - 1 \right]^2 \right\}^{\frac{1}{2}} \quad (27)$$

as the non-dimensionalized form of the desired relation for μ as

a function of β and δ . Similarly, by substituting (10) and (11) into (13), and making use of (21) and (24), one finds the relation for $\tan \delta$ in terms of α and μ .

$$\tan \delta = \frac{(1 + \mu \sin \alpha) \cos (\mu \cos \alpha) - 1}{(1 + \mu \sin \alpha) \sin (\mu \cos \alpha)} \quad (28)$$

By substituting (10), (11), and (21) into (12), one obtains β as a function of α and μ .

$$\beta = \left\{ [(1 + \mu \sin \alpha) \sin (\mu \cos \alpha)]^2 + [(1 + \mu \sin \alpha) \cos (\mu \cos \alpha) - 1]^2 \right\}^{\frac{1}{2}} \quad (29)$$

Taking the limit of $\tan \alpha$ from equation (22) as β goes to zero, while holding δ constant, leads to the indeterminate form $0/0$.

Hence, a MacLaurin series of the function $\tan \alpha$, expanded in terms of β , will be useful. This series is:

$$\tan \alpha = \tan \delta + \left(\frac{1 + \sin^2 \delta}{2 \cos \delta} \right) \beta + O(\beta^2) \quad (30)$$

Similarly, the MacLaurin series expansion of μ is:

$$\mu = \beta - \frac{1}{2}(\sin \delta \cos^2 \delta) \beta^2 + O(\beta^3) \quad (31)$$

where $O(\beta^2) \equiv$ terms of order β^2 and higher.

The out-of-plane angle σ in Cartesian coordinates is defined by

$$\tan \sigma \equiv \frac{y}{\rho} = \frac{y}{\sqrt{x^2 + z^2}} \quad \left(-\frac{\pi}{2} \leq \sigma \leq \frac{\pi}{2} \right) \quad (32)$$

By analogy, the out-of-plane curvi-angle ν is defined by

$$\tan \nu \equiv \frac{\nu}{\lambda} \quad \left(-\frac{\pi}{2} \leq \nu \leq \frac{\pi}{2}\right) \quad (33)$$

Using the fact that $\nu = y$, the last relationship can be written

$$\tan \nu = \frac{y/R}{\lambda/R} = \frac{y\rho/R\rho}{\mu} = \frac{\beta \tan \sigma}{\mu} \quad (34)$$

where the value of μ in terms of Cartesian-frame quantities is obtained from (27). The MacLaunrin series in β for $\tan \nu$ is obtained with the help of (31) as follows:

$$\tan \nu = [\tan \sigma] \left[1 + \frac{1}{2} (\sin \delta \cos^2 \delta) \beta + O(\beta^2)\right] \quad (35)$$

It will be noted from the series expansions that,

since $\beta \rightarrow 0$ as $\rho \rightarrow 0$,

$$\lim_{\rho \rightarrow 0} \alpha = \delta \quad \lim_{\rho \rightarrow 0} \mu = \beta \quad \lim_{\rho \rightarrow 0} \nu = \sigma \quad (36)(37)(38)$$

so that the curvilinear and Cartesian solutions are identical in an infinitesimal region about the origin.

The equations of this section hold when the relative state vector u, w lies in any quadrant of the reference coordinate system. The inverse tangent lies in the first, second, third, or fourth respectively as the sign of the numerator and denominator of its argument are $++$, $+ -$, $- -$, $- +$, respectively. Positive square roots are always selected.

COMPUTATIONAL ALGORITHM

This section incorporates the analytical results obtained in the last section into an algorithm for finding curvilinear state from a series of measurements. In general, either vehicle may make the required maneuver, take the measurements, and do the computation, provided the coordinate systems and sign conventions are set up properly. For definiteness, it is assumed that the measurements of the line-of-sight angles and the computations are done by the target, while the maneuver is made by the interceptor, the required information as to magnitude, direction, and time of occurrence being transmitted to the target through some suitable communication system. This corresponds, for example, to the nonmaneuvering Command Module (CM) being the target, and the maneuvering Lunar Module (LM) being the interceptor, with measurements of the LM being made by a sextant aboard the CM.

The principle used in this algorithm is that the measured line-of-sight angles δ and σ represent "hard" (i. e. accurate) data while the computed ranges ρ represent "soft" (i. e. less accurate) data. Hence, the δ 's and σ 's are never changed from the observed values during the iterative process. What is changed are the α 's and ν 's. The angles α and ν are obtained from the angles δ and σ by using the best range data

available (namely, the last set of ρ 's, or zero if no prior range data is available). These α - and ν -values may be thought of as synthetic measurements, that is, line-of-sight measurements in the curvilinear system. Then the curvilinear model is exercised to solve for the curvilinear state from the set of synthetic angle-measurements and the calibrated thrust maneuver. The curvilinear state thus obtained is geometrically transformed back to Cartesian coordinates, and a new set of ranges, more accurate than the last, is obtained. With better range data now available, a better set of synthetic angle-measurements can be obtained from the original δ 's and σ 's, and the process repeated.

As shown in Appendix A, state can be determined using the so-called 4-instant in-plane procedure by entering into the computation with a set of four $\tan \delta$ -measurements, $\tan \delta_1 \dots \tan \delta_4$ at times $\tau_1 \dots \tau_4$ plus a calibrated thrust maneuver at time τ_c^* . One can solve the Cartesian model to find the x, z state at each measurement time. This Cartesian state-determination is the first step to be executed, as shown in subroutine A of the flow chart of Fig. 2. In this step of the process, the horizontal and vertical components of the calibrated thrust maneuver (referred to the local horizontal at the interceptor) are designated $\Delta x'_c$ and $\Delta z'_c$ respectively. Next, the resulting Cartesian states $x_1, z_1 \dots x_4, z_4$ are entered into subrouting B, which

* This discussion which follows pertains to the four-instant in-plane procedure. It could just as easily be adapted to any method of determining state in Cartesian coordinates.

simply determines the in-plane range $\rho_1 \dots \rho_4$ at each measurement time. The values of $(\tan \delta)_i$ and ρ_i at each of the four times are next inserted into subroutine C, which converts each $\tan \delta$ to a $\tan \alpha$ by the use of Eq. (20) or Eq. (26). The four values of $\tan \alpha$, synthetic measurements of the line-of-sight curvi-angle, are entered, together with the corresponding times $\tau_1 \dots \tau_4$, into subroutine D. The horizontal and vertical components of the calibrated thrust maneuver are entered here as well, except that this time they are renamed $\Delta u'_c$ and $\Delta w'_c$ which is appropriate since they were defined as being horizontal and vertical at the interceptor's location. This is sufficient data to determine curvilinear state at the four measurement times using the four-instant in-plane computation mechanized in subroutine D. The output of this subroutine, $u_1, w_1 \dots u_4, w_4$ is entered into subroutine E, which transforms back to Cartesian coordinates. This is the second set of Cartesian states that has been generated. One is now ready to test to see if the answer is sufficiently accurate to terminate the computation. Subroutine F compares the first and second Cartesian solutions by differencing the two values of x and z at each of the four time points, squaring the differences, and summing them. If the root sum squares of the difference is less than some preset limit, the computation is completed. If it is not, then further work is in order. The next step, in the latter case, is to reenter Subroutine B, using the latest set of Cartesian states. The process

then continues around the loop until the test for convergence has been satisfied. (For safety, one normally would add a counter, terminating the computation after N iterations even if convergence is not obtained, with a special indication of that fact.) The last set of curvilinear states found by subroutine D are put out as the final result.

The reader may have recognized that subroutines A and D are essentially the same, the only difference being the names that are given to the inputs and outputs. Hence, while the flow chart of Fig. 2 is useful to understand the process, in actuality one would implement the computationally simpler flow chart shown in Fig. 3. The subroutines of Fig. 3 are essentially the same as those of Fig. 2, differences being indicated by the notes. In this mechanization, the calibrated thrust maneuver components are called only one name, $\Delta u'_c$ and $\Delta w'_c$ for the reason given above.

It is interesting to compare the curvilinear algorithm of Fig. 3 with a computerized algorithm for determining state in Cartesian coordinates. The latter requires only the subroutines B and E. From a computer storage point of view, the additional subroutines required to implement the curvilinear algorithm are C, E, and F, all of which are short. C and E are simple geometrical transformations from $\tan \delta$ to $\tan \alpha$ and curvilinear to Cartesian state, respectively. F is a simple bit of logic which directs the iteration process. From the computation time point of view, the Cartesian

solution is achieved in one pass through subroutine D. In the curvilinear solution, many passes are required, an apparent disadvantage of the curvilinear scheme. However, with the speed of modern guidance computers, the time required for as many passes as are required should present no real problem.

APPLICATION TO THE MANUAL METHOD

The equations given here for the curvilinear solution presume availability of a digital computer, whereas the Cartesian solution of Ref. 1 presumed manual computation. It is suspected that an attempt to compute the curvilinear solutions manually would overburden the astronaut. On the other hand, there is no reason why an astronaut could not use this method if he wanted to and had the time. The only new computations which the astronaut needs are (4), (10) and (11) which a MacLaurin series in u and w would simplify, and (22) or (30), already in the form of a MacLaurin series. The series are useful in that they tend to retain relatively high accuracy with comparatively few significant figures. If the astronaut were to make just one extra pass through the state-determination process, he could reduce the curvature error significantly. Of course, the curvature error at whort range is small enough without correction, so the astronaut should be advised to use this method only for ranges exceeding 50 nm.

SIMULATION STUDY

The algorithm presented here was tested in a digital computer simulation called CURVI-D. The set-up is described in this section; some of the results are presented in the next section. The simulation used the four-instant in-plane method of state-determination.

The target orbit (reference) was selected as a circular orbit with a 150 nm altitude over the earth's surface. In all cases, the calibrated thrust maneuver was made at time τ_3 . No errors other than curvature were assumed, and, in fact, the interceptor dynamics were presumed to be exactly described by equation (1). Instead of comparing the latest results with those of the prior iteration, the latest results were compared to true values, giving the exact value of the error in the state estimate. The iterative process was continued until the square root of the sum of the squares of the error in state at the four measurement times was less than one-thousandth of a nautical mile. Single precision FORTRAN was employed, using the exact formulas rather than the series expansions.

The variables which define the most general rendezvous problem of the class considered here are: a) initial conditions of interceptor relative to target at first measurement time, u_1, w_1, u'_1, w'_1 ; b) time interval from first to each of the second through fourth measurements; c) time of the calibrated thrust maneuver.

This gives a total of ten degrees of freedom. Obviously an exhaustive study over all variables would be prohibitively expensive. What was done was to choose a standard case, and then to explore various changes in conditions around this case. The standard case was the following: initial downrange distance $u_1 = 150$ nm, initial altitude $w_1 = 10$ nm, initial horizontal velocity such that a circular orbit was obtained (which implies that initial vertical velocity $w'_1 = 0$), initial time $\tau_1 = 0$, $\tau_2 = \pi/4$, $\tau_3 = \pi/2$, $\tau_4 = \pi$, $\tau_c = \tau_3$, a horizontal velocity $\Delta u'_c$ imparted which starts a Hohmann transfer to the target altitude, and a vertical velocity $\Delta w'_c = 0$. Then the variations about this case taken one at a time, were as follows: a) Range Study - variation of initial downrange distance from 50 to 250 nm; b) Size Study - variations in the size of the relative trajectory initial conditions (w_1 , u'_1 , w'_1) from half as large to twice as large; c) Size Study Including Burn - variation of the initial conditions and the calibrated thrust maneuver magnitude in a common ratio from 1/2 to 2 times the reference values; d) Horizontal Thrust Study - variation of $\Delta u'_c$ from -1 to $-\frac{1}{8}$ and from $\frac{1}{8}$ to 1 times the non-dimensional cost C/ω of the trajectory* of the standard case, ($C/\omega = 5$ on all cases, and $\Delta u'_c = -0.5 C/\omega$ on the standard

* As defined in Ref. 1, the cost of a trajectory in the relative coordinate system is equal to the sum of the ΔV of the two horizontal burns at apogee and perigee needed to drive the trajectory to rest on the x (or u) axis. It is the minimum ΔV which achieves this end condition.

case); e) Vertical Thrust Study - variation of the vertical component of the calibrated thrust maneuver from -1 to 0.2 and from 0.2 to 1 times C/ω , while setting the horizontal thrust component equal to zero, and f) Measurement Time Study - variation of the intervals $\tau_{21}, \tau_{31}, \tau_{41}$ uniformly by a factor varying from $\frac{1}{2}$ to 2 times the standard values. These six studies were performed on the standard case, which had a shape parameter r equal to zero⁽¹⁾. Then the initial conditions on w, u', w' were changed so that a shape parameter of $r = \frac{1}{2}$ was obtained, C/ω still being equal to 5. The complete set of six tests were then repeated on this set of initial conditions. Next, in turn, initial conditions on w, u', w' were altered so as to give initial shape parameters equal to 0, $-\frac{1}{2}$ and -1, and the six tests repeated for each shape. In all, 230 different rendezvous situations were simulated in this study.

Table I summarizes the initial conditions.

RESULTS

The results are shown in Figures 4-8. Interest in each of these cases centers on the following questions: a) did the run converge; b) by what factor was the RSS error in state reduced, on the average, in each iteration; and c) how many iterations were required to achieve the .001 nm RSS error? The convergence factor

is defined as the n^{th} root of the ratio of the RSS error on the first iteration to the RSS error on the last, where n is the number of iterations after the first. The convergence factor represents the average factor by which the error is reduced in each succeeding iteration. This factor is plotted on the figures (data points marked with circles), as is the Napierian logarithm of the number of iterations (data points marked with squares). Where a run converged to a non-nulled final error, the final error in nm is printed on the graph.

DISCUSSION OF RESULTS

The results show, in general, that the curvilinear algorithm is effective in nulling the curvature error. The convergence factor varies from slightly over one (it must be greater than one for convergence to occur) to more than 20, depending on the situation, with values between 5 and 10 being typical. The number of iterations (counting the first) varies from 4 to 572, with 6 to 16 being typical. Certain general trends are apparent. These serve as a guide to indicate what should be done to decrease the likelihood of having a non-converged case. First, it is evident from all five range studies that convergence is strongest at short downrange distances, growing weaker as range is increased. It appears that extension of this method to ranges longer than 250 nm is feasible, since the convergence factor in all five range studies was still

TABLE I

INITIAL CONDITIONS FOR REFERENCE CASES

Fig. No.	r	Cost $\frac{\omega}{\omega}$	Average Altitude	Amplitude of w-Oscillation	u_1	w_1	u'_1	w'_1	w-max	w-min
4	1	5	10	0	150	10	-15	0	10	10
5	$\frac{1}{2}$	5	10	$\frac{10}{3}$	150	10	-15	$\frac{10}{3}$	$1\frac{1}{3}$	$2\frac{2}{3}$
6	0	5	10	10	150	10	-15	10	20	0
7	$-\frac{1}{2}$	5	$\frac{10}{3}$	10	150	10	$-18\frac{1}{3}$	$\frac{10\sqrt{5}}{3}$	$1\frac{1}{3}$	$2\frac{2}{3}$
8	-1	5	0	10	150	10	-20	0	10	-10

Distances given in nm; velocities in nm/rad.

greater than 2 at this range. Secondly, from the size study in which the calibrated thrust maneuver was varied in proportion to the initial conditions on w , u' , and w' that the larger the relative trajectory, the higher the convergence factor. Yet even with the smallest size relative trajectory studie (5 nm initial altitude separation), no difficulty in converging was experienced. The horizontal and vertical thrust studies shows similar trends -- the larger the burn, the larger the convergence factor. A burn which is too small may result in non-convergence, or convergence to an erroneous state (see Figs. 4 and 5). The measurement time studies show that short intervals between measurements are better than long ones for convergence.

In the 230 cases studied, all but three converged to correct final states. Insufficient ΔV in the calibrated thrust maneuver was responsible for these three. It appears that, irrespective of direction, if the magnitude of the burn is large enough, convergence is obtained. In the cases studied, a burn whose magnitude was equal to or larger than 25% of the cost (Ref. 1) of the initial trajectory gave convergence. (It may be that the percentage figure quoted above is itself a function of initial downrange distance, this particular value being applicable for 150 nm).

It is likely that rendezvous would be achieved even in the three cases which converged to an erroneous state, if the guidance scheme of Ref. 1 were used, because of the servo-like ability of the guidance loop to null out the effects of navigation errors, as demonstrated in Ref. 3.

CONCLUSIONS

A simple algorithm has been presented for deterministically generating an estimate of relative state in the high-accuracy curvilinear coordinate system, using any of the state-determination methods previously developed in Ref. 1 for Cartesian coordinate systems. Simulation results demonstrate the effectiveness of the method for in-plane state-determinations out to ranges of 250 nm.

RECOMMENDATIONS FOR FURTHER WORK

This simulation should be extended to test out-of-plane motion and maneuvers. Improvement of convergence in the marginal and/or erroneous cases may be a by-product of such an extension. The method should also be tested in cases with finite eccentricity of the reference orbit. In this connection, the techniques of the Local Tangent Coordinate System, as reported in Ref. 5, should prove useful.

SYMBOLS^{*}

u, w	relative downrange distance and altitude in in curvilinear coordinates.
u', w'	derivative of u, w with respect to τ .
α	$\tan^{-1} w/u$, in-plane line-of-sight curvi- angle.
ϕ	central angle subtended by u .
R	radius of circular reference orbit.
ω	mean motion of circular reference orbit.
β	ρ/R
μ	λ/R
λ	$\sqrt{u^2 + w^2}$, in-plane curvi-range.
subscripts 1, 2, 3, 4	refer to times $\tau_1, \tau_2, \tau_3, \tau_4$ at which meas- urements are taken.
subscript c	refers to the time of the calibrated thrust maneuver.
$\Delta u'_c, \Delta w'_c$	magnitude of local horizontal and vertical components of a calibrated thrust maneuver made at τ_c .
Ψ_{ijkl}	the i, j^{th} element of $\Psi(\tau_{k\ell})$
ν	$\tan^{-1} y/\lambda$, out-of-plane line-of-sight curvi- angle

* Other than those of Ref. 1.

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APPENDIX A

The Four-Instant In-Plane State Determination Algorithm

An algorithm is developed for determining in-plane state in a Cartesian coordinate system based on four measurements of the in-plane line-of-sight angle δ at times $\tau_1, \tau_2, \tau_3, \tau_4$ and a burn $\Delta x'_3$ and $\Delta z'_3$ immediately following the third measurement, i. e. at τ_{3+} .^{*†} It is shown in Ref. 1 that the nondimensional horizontal and vertical velocity at τ_3 , ξ_3 and η_3 , defined as follows:

$$\xi_3 \equiv \frac{x'_3}{z_3} \qquad \eta_3 \equiv \frac{z'_3}{z_3} \qquad (A-1)$$

are obtainable from the computations:

$$\xi_3 = \frac{1}{D} (C_{13} B_{23} - C_{23} B_{13}) \qquad (A-2)$$

$$\eta_3 = \frac{1}{D} (A_{13} C_{23} - A_{23} C_{13}) \qquad (A-3)$$

where

$$D = A_{13} B_{23} - A_{23} B_{13} \qquad (A-4)$$

*Notation and nomenclature follow Ref. 1.

†Measurements are assumed to be made by a sextant. If a reference x-direction is available, as from an inertial reference package, then δ can be measured directly. Otherwise, δ can be computed from a measurement of the angle between the line-of-sight to the vehicle and the line-of-sight to each of two identified stars, providing, as is always postulated, that the celestial direction of the reference axis x is known at each measurement time.

$$A_{ji} = (4s_{ji} - 3\tau_{ji}) \tan \delta_j + 2(c_{ji} - 1) \quad (A-5)$$

$$B_{ji} = 2(c_{ji} - 1) \tan \delta_j - s_{ji} \quad (A-6)$$

$$C_{ji} = 4 - 3c_{ji} - 6(s_{ji} - \tau_{ji}) \tan \delta_j - \frac{\tan \delta_j}{\tan \delta_i} \quad (A-7)$$

In these equations, the following notation and definitions apply:

$$s_{ji} \equiv \sin \tau_{ji}$$

$$c_{ji} \equiv \cos \tau_{ji}$$

$$\tau_{ji} \equiv \tau_j - \tau_i$$

$$\tau \equiv \omega t$$

$$t \equiv \text{time}$$

$$\omega \equiv \text{mean motion of the reference orbit}$$

$$x' \equiv \frac{dx}{d\tau} \equiv \frac{1}{\omega} \frac{dx}{dt}$$

Using the convention that $[x_3, z_3, x'_3, z'_3]^T$ is the in-plane state vector immediately prior to the burn, the following equation gives the state at τ_4 as it is propagated from state at τ_3

$$\begin{bmatrix} x \\ z \\ x' \\ z' \end{bmatrix}_4 = \Psi_{43} \begin{bmatrix} x \\ z \\ x' \\ z' \end{bmatrix}_3 + \Psi_{43} \begin{bmatrix} 0 \\ 0 \\ \Delta x'_3 \\ \Delta z'_3 \end{bmatrix} \quad (A-8)$$

This equation is the specialization of (1) for in-plane state, the 4×4 matrix Ψ being the appropriate rows and columns of Φ .

Dividing (A-8) by z_3 , and using the measurement of δ_3 and the definition (A-1)

$$\begin{bmatrix} x_4/z_3 \\ z_4/z_3 \\ x'_4/z_3 \\ z'_4/z_3 \end{bmatrix} = \Psi_{43} \begin{bmatrix} \text{ctn } \delta_3 \\ 1 \\ \xi_3 \\ \eta_3 \end{bmatrix} + \Psi_{43} \begin{bmatrix} 0 \\ 0 \\ \frac{\Delta x'_3}{z_3} \\ \frac{\Delta z'_3}{z_3} \end{bmatrix} \quad (\text{A-9})$$

Writing out the full expressions for the first two elements of the left side yields

$$\begin{aligned} \frac{x_4}{z_3} = & \Psi_{1143} \text{ctn } \delta_3 + \Psi_{1243} + \Psi_{1343} \xi_3 + \Psi_{1443} \eta_3 \\ & + \Psi_{1343} \frac{\Delta x'_3}{z_3} + \Psi_{1443} \frac{\Delta z'_3}{z_3} \end{aligned} \quad (\text{A-10})$$

$$\begin{aligned} \frac{z_4}{z_3} = & \Psi_{2143} \text{ctn } \delta_3 + \Psi_{2243} + \Psi_{2343} \xi_3 + \Psi_{2443} \eta_3 \\ & + \Psi_{2343} \frac{\Delta x'_3}{z_3} + \Psi_{2443} \frac{\Delta z'_3}{z_3} \end{aligned} \quad (\text{A-11})$$

The symbol Ψ_{1243} denotes the 1, 2 element of the Ψ matrix, evaluated for the interval τ_{43} .

Dividing (A-11) by (A-10), and using the definition of δ_4 ,

$$\tan \delta_4 = \frac{\Psi_{2143} \operatorname{ctn} \delta_3 + \Psi_{2243} + \Psi_{2343} \xi_3 + \Psi_{2443} \eta_3 + \Psi_{2343} \frac{\Delta x'_3}{z_3} + \Psi_{2443} \frac{\Delta z'_3}{z_3}}{\Psi_{1143} \operatorname{ctn} \delta_3 + \Psi_{1243} + \Psi_{1343} \xi_3 + \Psi_{1443} \eta_3 + \Psi_{1343} \frac{\Delta x'_3}{z_3} + \Psi_{1443} \frac{\Delta z'_3}{z_3}} \quad (\text{A-12})$$

Cross-multiply and transpose so that only velocity terms appear on the left,

$$\begin{aligned} (\Psi_{1343} \tan \delta_4 - \Psi_{2343}) \frac{\Delta x'_3}{z_3} + (\Psi_{1443} \tan \delta_4 - \Psi_{2443}) \frac{\Delta z'_3}{z_3} &= \Psi_{2143} \operatorname{ctn} \delta_3 + \Psi_{2243} + \\ &+ \Psi_{2343} \xi_3 + \Psi_{2443} \eta_3 - (\Psi_{1143} \operatorname{ctn} \delta_3 + \Psi_{1243} + \Psi_{1343} \xi_3 + \Psi_{1443} \eta_3) \tan \delta_4 \quad (\text{A-13}) \end{aligned}$$

Solve for z_3 ,

$$z_3 = \frac{(\Psi_{1343} \tan \delta_4 - \Psi_{2343}) \Delta x'_3 + (\Psi_{1443} \tan \delta_4 - \Psi_{2443}) \Delta z'_3}{\Psi_{2143} \operatorname{ctn} \delta_3 + \Psi_{2243} + \Psi_{2343} \xi_3 + \Psi_{2443} \eta_3 - (\Psi_{1143} \operatorname{ctn} \delta_3 + \Psi_{1243} + \Psi_{1343} \xi_3 + \Psi_{1443} \eta_3) \tan \delta_4} \quad (\text{A-14})$$

The matrix elements are⁽¹⁾

$$\Psi_{11kl} = 1$$

$$\Psi_{12kl} = 6(s_{kl} - \tau_{kl})$$

$$\Psi_{13kl} = 4s_{kl} - 3\tau_{kl}$$

$$\Psi_{14kl} = 2(c_{kl} - 1)$$

$$\Psi_{21kl} = 0$$

$$\Psi_{22kl} = 4 - 3c_{kl}$$

$$\Psi_{23kl} = 2(1 - c_{kl})$$

$$\Psi_{24kl} = s_{kl}$$

Equation (A-14) is used to generate z_3 from the knowledge of δ_3 , δ_4 , $\Delta x'_3$, $\Delta z'_3$, ξ_3 , η_3 , and the elements of the state-transition matrix. Once z_3 is known, the other in-plane state elements at τ_3 are immediately available as follows:

$$x_3 = z_3 \operatorname{ctn} \delta_3 \quad (\text{A-15})$$

$$x'_3 = \xi_3 z_3 \quad (\text{A-16})$$

$$z'_3 = \eta_3 z_3 \quad (\text{A-17})$$

Having all of the state elements at τ_3 , the state at τ_1 and τ_2 is obtained using (1) for the appropriate time interval, and state at τ_4 is obtained using (A-8).

These equations are equally valid using curvilinear coordinates when substitutions are made according to the following table:

<u>Cartesian</u>	<u>Curvilinear</u>
x	u
z	w
x'	u'
z'	w'
$\Delta x'$	$\Delta u'$
$\Delta z'$	$\Delta w'$
$\xi = x'/z$	$\gamma = u'/w$
$\eta = z'/z$	$\zeta = w'/w$
$\delta = \tan^{-1} z/x$	$\alpha = \tan^{-1} w/u$

The generalization of these equations to the case where the calibrated thrust maneuver occurs at some time other than τ_3 is straightforward.

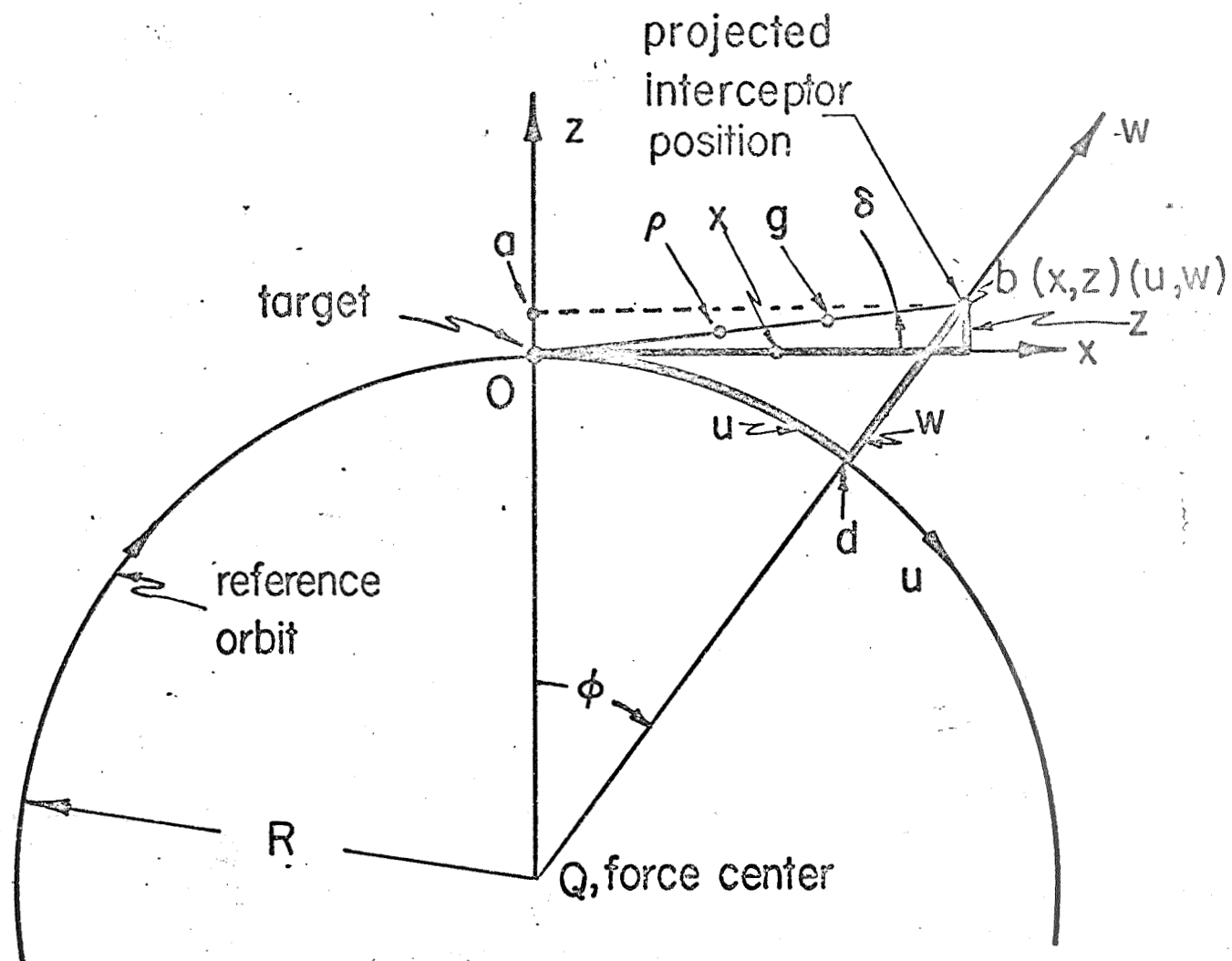
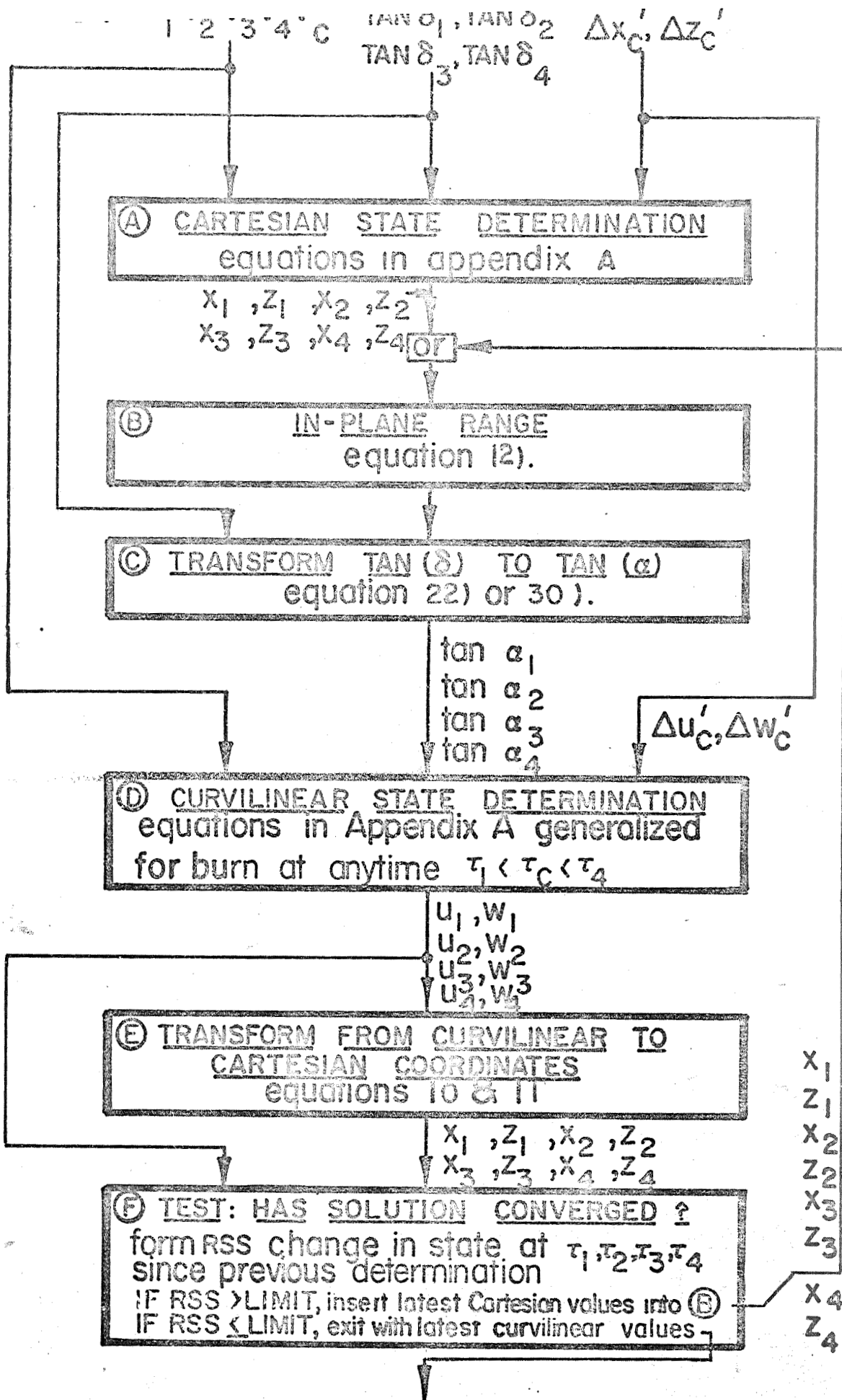


Fig. I. Cartesian and Curvilinear Coordinate Systems



OUTPUTS: $u_1, w_1, u_2, w_2, u_3, w_3, u_4, w_4$

Fig. 2. Flow Chart for Determining State in Curvilinear Coordinates.

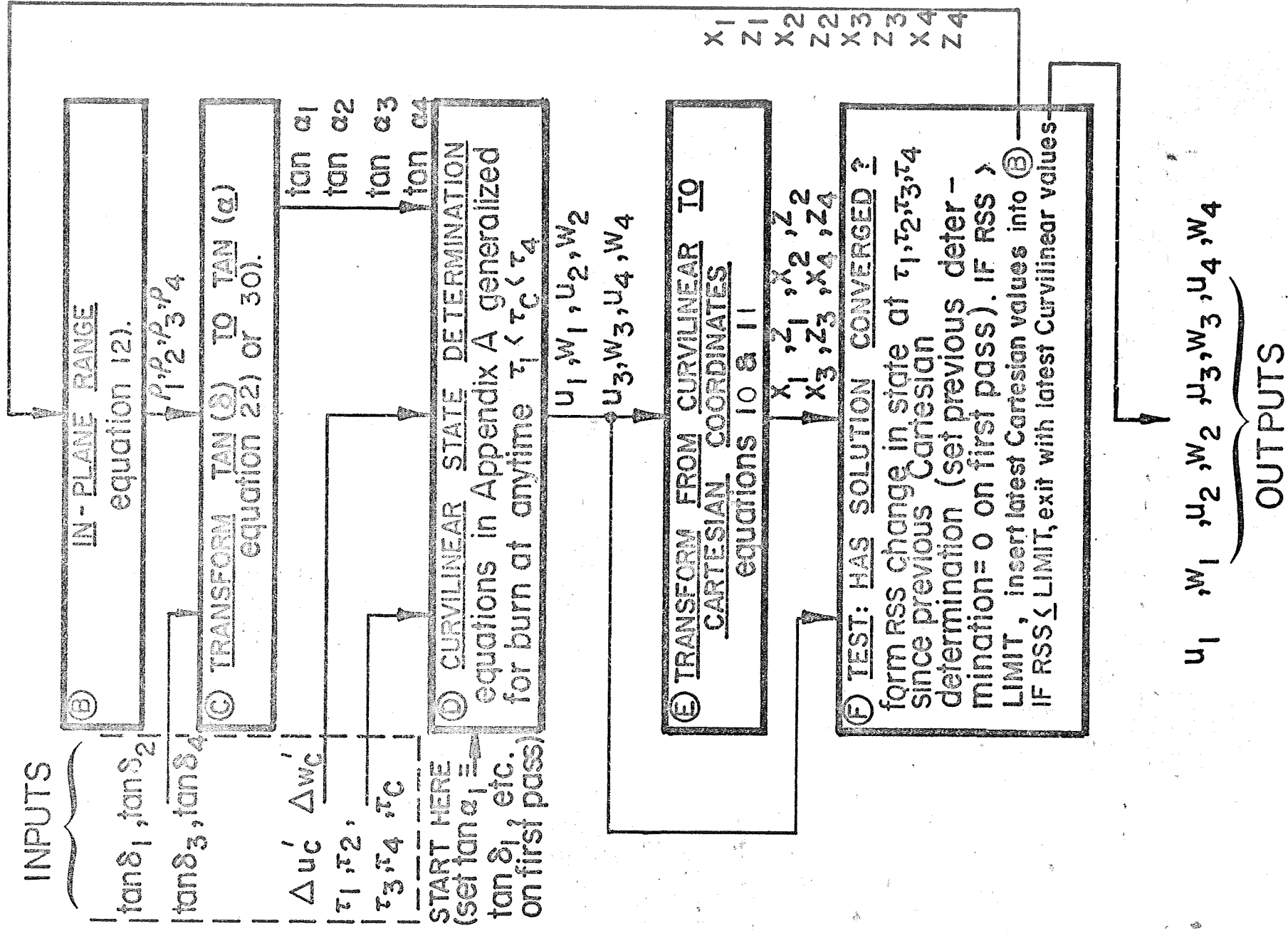


Fig. 3 Simplified Flow Chart

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An astronaut and computer work together to simulate a manual rendezvous system, which is shown to achieve the desired mission in spite of realistic, error-induced perturbations.

Summary—A system for navigation, guidance, and control of a spacecraft to rendezvous with an orbiting target, based entirely on observations by handheld, unpowered instruments, and computations done entirely by hand, has been developed. This paper describes results of an interactive simulation of this system through a selected set of rendezvous missions, demonstrating feasibility in the presence of realistic errors. The simulation is carried out by a trained operator, the "astronaut", working in conversational mode with a desk-size digital computer. The computer represents the environment, and the operator represents the astronaut's decision-making process. Plots are given showing the increase in fuel and in rendezvous time as a function of the magnitude of the principal error sources, namely: model error due to separation range, model error due to eccentricity of the reference orbit, control errors in direction and magnitude of burn vectors, and measurement error. It is shown that the manual rendezvous system is strongly convergent, so that the real penalty of errors is not failure to rendezvous, but rather, using more fuel and/or time than would otherwise have been necessary.

NOTATION

A. Symbols

A	coefficient matrix of differential equation of motion
c	$\cos \tau$
Cost	combined fuel-time cost criterion
k	cost-of-time weighting constant, knots/rad.
K	scaling factor for linearized model
r	shape parameter of relative trajectory

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s	$\sin \tau$
t	time
T	period of reference orbit
ω	angular velocity of reference orbit, rad./hr
$\vec{\omega}$	angular velocity vector of reference orbit
x	downrange component of relative displacement, nm
x', y', z'	relative trajectory velocity components, i.e. derivative of x, y, z with respect to τ
X	6×1 state column vector of position and velocity
y	out-of-plane component of relative displacement, nm
z	upward component of relative displacement, nm
z_a	the smaller in absolute value of the two extrema of z along relative trajectory
z_b	the larger in absolute value of the two extrema of z along relative trajectory
δ	in-plane angle (angle between x -axis of LTCS and projection of LOS onto plane of reference orbit)
ΔV	velocity change magnitude, knots
e	eccentricity
η	dimensionless vertical velocity, z'/z
ξ	dimensionless horizontal velocity, x'/x
σ	OOP angle (angle between LOS and its projection onto the reference orbit plane)
τ	normalized time (rad. of travel in reference orbit), ωt
$\bar{\tau}$	dimensionless time interval (rad.) between a point on a trajectory and nearest point at which $z=z_b$

- Φ state transition matrix of linearized equations of motion
- [3] 3 instant out-of-plane state determination
- [4] 4 instant in-plane state determination

B. Abbreviations

A_p	apogee passage
2B	two-burn maneuver calculation
3B	three-burn maneuver calculation
DB	dark period begins
DE	dark period ends
hr	hours
IP	in-plane
IPNS	in-plane normalized state
LOS	line-of-sight
LTCS	local tangent coordinate system
mr	milliradians
nm	nautical miles
OOP	out-of-plane
PC	plane-crossing time
P_e	perigee passage time
rad.	radians
SP	state projection
t_c	plane-crossing time
t_e	extremum time

C. Other notations

- ($\cdot$) differentiation with respect to t
- ($\cdot$)' differentiation with respect to τ
- 0 subscript, initial conditions
- $\equiv$ equals by definition
- measurement
- $\otimes$ out-of-plane thrust maneuver
- $\bigcirc \rightarrow$ in-plane thrust maneuver

INTRODUCTION

A SYSTEM for navigation, guidance, and control of a spacecraft to rendezvous with an orbiting target, based entirely on observations by handheld, unpowered instruments, and computations done entirely by hand, has been developed [1]. This system uses an optical sextant developed by the Air Force for use in space [2]. The system was designed in cooperation with the developers of the sextant, and was made to be compatible with a scheme they had worked out for orbit determination [3]. A prototype model of the sextant was flown and space-tested in the Gemini IV and VII flights, the results showing it to possess more than sufficient accuracy for the purpose for which it was developed [4].

This present paper describes results of an interactive simulation of the manual guidance system through a selected set of rendezvous missions, together with results of studies of the principal sources of error.*

* This research is being supported by NASA under Grant No. NGR-05-009-106.

The simulation is carried out by a trained operator, the "astronaut", working in a conversational mode with a desk-size digital computer. The computer keeps track of the exact position of target and spacecraft relative to inertial space, and to each other. It implements spacecraft maneuvers called for by the astronaut, suitably changing the orbital parameters of the spacecraft. It keeps track of light and dark periods, assuming the target to be visible only in the dark for purposes of sextant observations. It maintains the "star field", i.e. celestial coordinates, and gives the astronaut the sighting angles that would be observed between the line-of-sight to the target and the line-of-sight to selected stars.

The astronaut makes the human decisions. He decides the number of sightings to be made and the time at which each is taken. By interpreting the results of his sightings according to the manual guidance system, he decides what kind of flight plan to implement—the direction, magnitude, and time of each burn—and then "commands" each burn by notifying the computer of his decision. Working close to real time, he leads the computer through the complete rendezvous mission. In short, the computer represents the environment, and the operator represents the astronaut's decision-making processes.

Appendix A summarizes the essential features of the manual rendezvous system.

This study seeks to evaluate the manual system through a numerical analysis of the principal error sources, namely, linearized model error due to finite separation range; model error due to eccentricity of the reference orbit; control errors in direction and magnitude of burn vectors; and measurement error including sextant, window and operator contributions. Data on each of the error sources is obtained from digital simulations of a series of complete rendezvous missions in which only a single error source is allowed to act at an increasing level of severity. Then, as a final test, selected missions are carried out with anticipated realistic values of several error sources acting concurrently. The evaluation is felt to be reasonably representative of what should be obtainable in practice. It will be shown that the real penalty of errors is not failure to rendezvous, but rather, greater use of fuel and/or time than would otherwise have been necessary.

SIMULATION SET-UP

Digital simulation of the manual rendezvous scheme is performed on an IBM 1130 computer, a desk-size machine featuring internal disk storage. This computer was chosen because (1) its dollar cost per hour is low enough to be feasible for completely dedicated use by a human operator whose reaction time is significant, and (2) its input-output

lends itself to the kind of operation desired, i.e. insertion of an astronaut decision, computation, output of a computational result reflecting a change in the environment, evaluation of the result by the astronaut and finally, a new astronaut decision, repeating the process. The structure of the FORTRAN program used in the simulation is designed to implement this conversational mode. The program is best described as consisting of a main intercommunication section which interfaces with more than 20 subroutines for the basic calculations.

In short, the computer is only a tool which the operator uses to simplify his tasks. In the simulation, as in the actual mission, the astronaut must achieve the rendezvous.

A. Computer functions

The computer program is divided into three major groups of functions. One group of subroutines simulates the environment, or "real world". Initial conditions are specified to the program by inserting the state vector, position and velocity, of each vehicle in absolute coordinates at the time of target orbit perigee passage. Time is arbitrarily set to 0 hr at this instant. Given these initial state vectors, the real world portion of the program propagates the two orbits in inertial space over a spherical earth with inverse-square force field using the Keplerian equations of motion. The measurement process is simulated by the operator's specifying the time at which a real astronaut would actually use his sextant. The computer updates the position of both vehicles relative to the stars to the selected time. As a time-saver, instead of calculating the direct sextant angles from such a measurement, the program goes one step further and computes the line-of-sight angles σ and δ as seen in the LTCS.* Since it is assumed for this simulation that the astronaut can only take sextant measurements when the target is in the dark (this is probably overly conservative) the real world simulation keeps track of the relation of the target vehicle to the light and dark portions of its orbit.

A second major group of subroutines carry out the "manual computations", i.e. those which the astronaut would normally perform using the charts, tables, and graphical procedures outlined in Ref. [1]. The reason they have been computerized is to save the operator's time in conducting simulated missions. In totality, they are functionally equivalent to the manual computations, producing the same outputs for a given set of inputs.

The third group of computer functions handles bookkeeping operations and simplifies the task of evaluating simulation results. For example, the computer keeps track of the true relative state

vector to use as a reference against which the astronaut's estimate can be compared, and errors determined. Careful distinction is always made between the data legitimately available to the astronaut and data available only to the operator in his role as designer and evaluator of the system.

B. Operator's functions

In the role of astronaut, the operator simulates all of the human decisions which would be made in a real mission regarding the navigation, guidance and control of his vehicle in relation to the target. He decides (1) the time of sextant readings (2) the magnitude, direction and time of all burns (3) the method of state determination (4) the type of flight plans to be used and (5) the conditions under which the rendezvous mission is terminated. In making these decisions, he normally follows the procedures of the manual system, using the data obtained in his previous calculations. However, he is entirely free at any time to disregard the standard procedures and rely on his physical intuition and the insight gained from experience in using the manual rendezvous technique. Although the decision-making time used by the operator in a mission is nearly equal to that of an astronaut, computerization of all calculations and instantaneous update of trajectories over specified time intervals allows the former to achieve rendezvous in $\frac{1}{4}$ – $\frac{1}{3}$ the time it would actually take in real life.

DESCRIPTION OF TEST CASES

For the purposes of this study, three test cases were selected. Since the manual rendezvous method is perfectly general and capable of rendezvous from any initial condition, provided only that the target remains visible until the first state-determination, the cases were chosen to cover a wide span among the total range of possibilities. To completely define initial conditions for a given case, one chooses (1) the initial relative state vector in the LTCS (2) the time at which the first dark period begins (3) the reference orbit eccentricity, ϵ (4) the period of the reference orbit, T^* and (5) the cost-of-time weighing constant, k . As an example, initial conditions for Case 1 are: $x_0 = -0.1999$, $y_0 = 0.0095$, $z_0 = -0.01$, $\dot{x}_0 = 0.061$, $\dot{y}_0 = 0.0123$, $\dot{z}_0 = 0.0$ [distances in nautical miles (nm) and velocities in nm/rad.]; dark begins at 0 hr; $\epsilon = 0$; $T = 1.5697$ hr (corresponds to 260 nm altitude); and $k = 0$.

Figure 1 shows the absolute and relative trajectory in the LTCS for each case in the absence of thrusting.

* For definitions of σ , δ and LTCS, see Appendix A.

* The orbital period of one vehicle or the other must be known by an astronaut desiring to use the manual method. This can also be found by manual means [3].

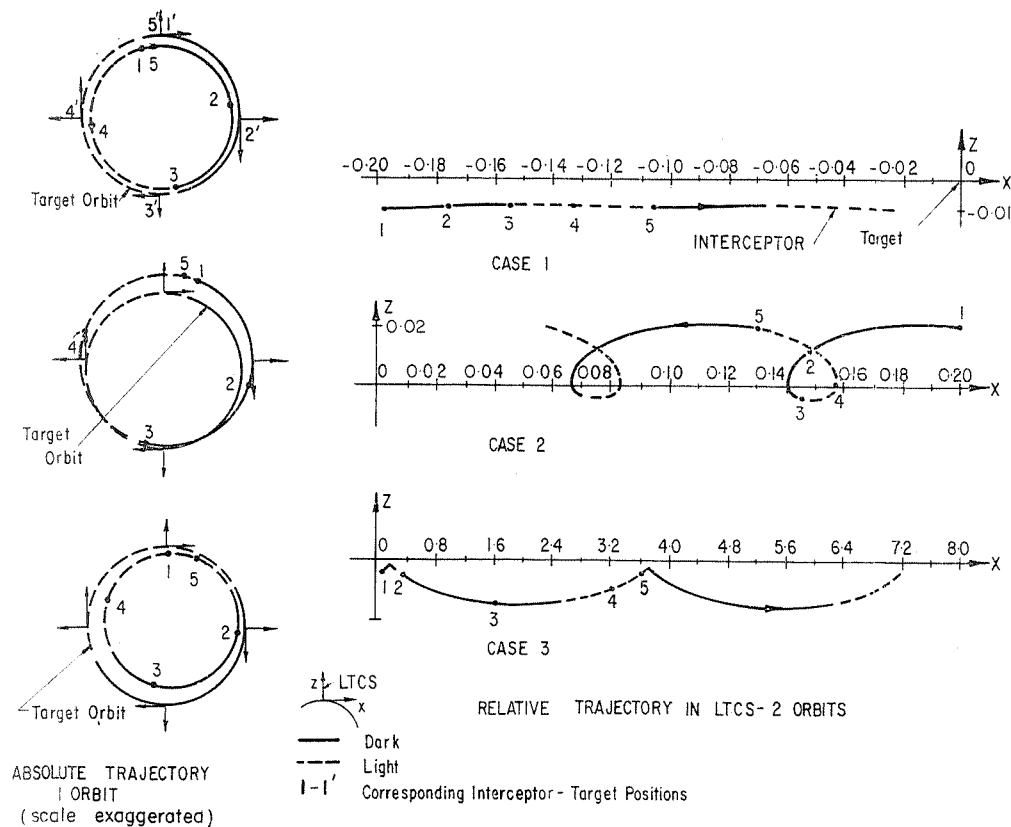


FIG. 1. Test case initial conditions.

Case 1 is typical of the situation in which the interceptor vehicle is launched to rendezvous with a target whose orbit is reasonably well known. It is placed in good position for rendezvous, at a lower altitude and slightly behind the target. The fact that the relative interceptor trajectory is nearly, but not quite, coelliptical, i.e. it is nearly a straight line as seen in LTCS, is a reflection of an assumed slight uncertainty in the target orbit parameters by the system that established the interceptor in its present situation. Case 2 was chosen to illustrate the capability of the method for achieving rendezvous when the astronaut's altitude before maneuvering was sometimes higher and sometimes lower than the target vehicle. Case 3 was designed to test the full capability of the method under extremely, and perhaps unrealistically, difficult conditions, the magnitudes for each component of the initial relative state vector having been chosen at random. For brevity, only the results of Case 1 will be presented in detail.

STRATEGIES FOR THE TEST CASES

Before proceeding, it is necessary to define pragmatically what constitutes an "error-free" mission on the digital simulator. The criteria of Table 1 were used throughout the study.

The date given in the preceding section for Case 1 establishes initial conditions for an error-free

trajectory, with respect to both separation range and reference orbit eccentricity for that case.

TABLE 1. STANDARD CONDITIONS FOR "ERROR-FREE" MISSIONS

Error source	Magnitude
Separation range	0.2 nm at initial time—this short range effectively nulls the model error in the linearized equations due to finite range
Eccentricity of reference orbit	The target orbit is circular ($e=0$) with radius being 3700 nm—this nulls the model error in the linearized equations due to eccentricity
Control errors in burn vector	Magnitude—accurate to nearest 0.00005 knot* Direction—accurate to nearest 0.00005 rad.*
Measurement error	Line-of-sight angles σ and δ accurate to nearest 0.00005 rad.*

* These values establish the level of round-off error of the operator in entering data.

In the remaining sections of this paper, the rendezvous mission which satisfies the rules of Table 1 will be known as the reference mission of the case. The test plan calls for evaluating the principal error sources one by one. A given error-producing effect is introduced into a rendezvous mission that has all other error sources held at the reference mission values, and a new rendezvous mission is run with this error source. A series of such runs is made, with the error source set successively at higher values. The successive missions related to the reference mission will be denoted by the magnitude of the individual error source present, e.g. the 10-mile range mission, the 0.03 eccentricity mission, etc.

To achieve rendezvous from a given set of initial conditions requires the implementation of a strategy—the particular sequence of measurements and the kind(s) of flight plans used by the astronaut during a mission. In principle, it is possible to determine an optimum strategy for each mission separately. However, in this study, in order to evaluate the influence of each individual error source, a common strategy was normally used for all missions of the test case regardless of the magnitude or type of error present. Several strategies were investigated before one was selected which would represent a reasonable approximation to an optimal strategy for all related missions for the case. The use of one common strategy also makes it possible to compare the error sources to determine their relative influence on the manual rendezvous scheme. This comparison would be less meaningful if different strategies were used for successive missions. Table 2 presents the astronaut's reasoning and the resulting strategy for Case 1. The operator achieves rendezvous with the target when both:

Final separation range $\leq 0.01 \times (\text{maximum separation range})$

Final velocity magnitude (knots) $\leq 4 \times (\text{numerical value of final separation range in nm})$

TABLE 2. STRATEGY—CASE 1

	Event	Time (hr)	Reasoning
1.	DB	0.0	Dark begins
2.	●	0.0001 0.2 0.4	These measurements will determine the times of astronaut orbit perigee-apogee passage and the orbital plane-crossing time
3.	⊗	PC*	Calibrated thrust removing approximately $\frac{1}{2}$ remaining out-of-plane velocity magnitude. Residual velocity is purposely left to aid in future state determinations

Table 2 (continued)

	Event	Time (hr)	Reasoning
4.	●	0.6	A measurement after the out-of-plane thrust is made to set up state determination using [3]
5.	[3]		Measurements at 0.2, 0.4, 0.6 and out-of-plane thrust are used to determine state at 0.4 hr
6.	SP		State is projected to the apogee passage time where a possible in-plane thrust could be made. The astronaut sees he is making good progress and decides to coast for at least one full orbit
7.	●	0.78	Measurement taken just before leaving the dark. It is good practice to take extra measurements in case a previous measurement should turn out to be faulty
8.	DE	0.7848	Dark ends
9.	SP	1.0 1.2 1.4	The known state at 0.4 hr is projected forward to monitor progress during the light period
10.	DB	1.5698	
11.	●	1.57 1.76 1.95	See Event 2
12.	⊗	PC	See Event 3
13.	●	2.15	See Event 4
14.	[3]		Measurements at 1.76, 1.95, 2.15 and out-of-plane thrust are used to redetermine state at 1.95 hr. Since the separation range is reduced, this determination should be more accurate than the one at Event 5
15.	SP		State is projected to apogee passage time. Astronaut sees that waiting one more orbit would put him ahead of target, an undesirable situation. He decides to initiate a 2-burn maneuver putting him into a coelliptic orbit with smaller altitude differential from target
16.	○→	A_p	First burn of maneuver
17.	●	2.35	See Event 7
18.	DE	2.3546	
19.	SP	2.55 2.75 2.95	See Event 9 (known state at 1.95 hr)
20.	○→		Second burn of maneuver. This burn should establish the interceptor in a coelliptical ($r=1$) orbit
21.	DB	3.1394	

* The time of this maneuver is the astronaut-calculated plane-crossing time, which differs slightly from one mission to the next.

Table 2 (continued)

	Event	Time (hr)	Reasoning
22.	●	3-14 3-33 3-52	See Event 2
23.	⊗	PC	See Event 3
24.	●	3-75	See Event 4
25.	3		Measurements at 3-33, 3-52, 3-75 and out-of-plane thrust are used to redetermine state at 3-52 hr. The 4 procedure using the burn entering the parking orbit could have been used but there was sufficient time for the more accurate 3 procedure
26.	SP		State at 3-52 hr is projected to the time of the first burn of Hohmann transfer
27.	●	3-92	See Event 7
28.	DE	3-9242	
29.	2B		Calculations of the burn vector components for the Hohmann transfer
30.	○→		The first burn of a two-burn transfer is made when astronaut's estimated state crosses the Hohmann line†
31.	SP	4-15 4-35 4-55	See Event 9 (known state at 3-52 hr)
32.	⊗	PC	All remaining out-of-plane velocity is removed at this plane crossing time which was determined from the previous state. The astronaut assumes he will not need another state determination
33.	DB	4-7090	
34.	○→		Second burn of transfer at $\frac{1}{2}$ orbit after previous burn. Interceptor is now in favorable position for docking

† The line from which a Hohmann transfer causes an interceptor in a coelliptic orbit to meet the target.

Figure 2 shows the use of the strategy on the reference mission. The symbols for major events used in this figure and in Table 2 are given in the Notation.

Case 1 (see Fig. 2)

The interceptor is in a nearly coelliptic orbit, behind, and slightly below the target. Rather than attempting to rendezvous in minimum time, the astronaut decides to minimize fuel only. Hence, $k=0$ for this case. He decides to adopt a policy of coellipticity* that will ensure a sequence of very good state determinations throughout the mission. After in-plane coasting for $1\frac{1}{2}$ orbits, a two-burn maneuver places his vehicle in a coelliptic parking orbit with a small differential altitude. From this situation, an accurate state determination is made verifying his new orbit. A simple Hohmann transfer completes the mission.

PROCEDURE

With the magnitudes of the principal error sources set at the "error-free" conditions of Table 1, the complete reference mission for the test case was simulated and data was obtained on both the total fuel expenditure, as measured by ΔV , and time in achieving rendezvous. The following results were obtained: total $\Delta V=0.0594$ knots, rendezvous time=5.09 hr. Then, a given error-producing effect was introduced, the test case re-run with all other error sources held at the reference mission values, and new results on fuel and time obtained. A series of such rendezvous missions was simulated with the magnitude of the error source of interest set at successively higher values. Any velocity remaining when the rendezvous criterion is met is added, using the "cost of a trajectory" formula [1], to the ΔV spent in burns, to arrive at the total ΔV for the mission. The excess ΔV and rendezvous time required in a case with

* Coelliptic orbits are not specifically included in the present formulation of the Flight Planning Charts of the manual method.

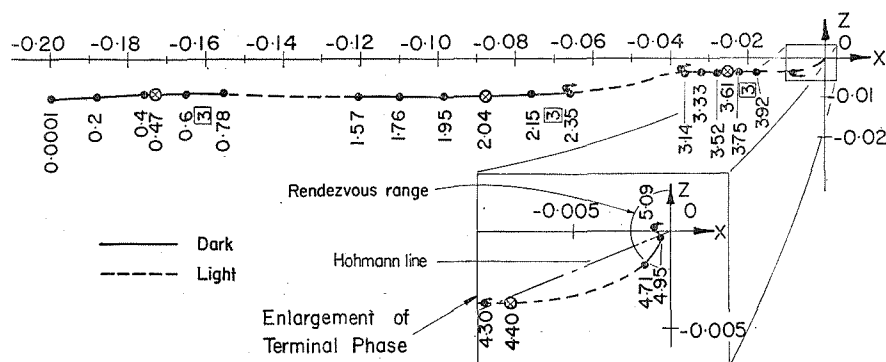


FIG. 2. Reference trajectory—Case 1.

error present is an indication of the mission penalty which such an error imposes.

Model error due to separation range was introduced by scaling the initial relative state vector of the test case. If there were no model error, the only effect upon the reference mission data would be an increase in fuel in proportion to the scale factor. The time to achieve rendezvous would be constant since at any instant the state vector of the separation range mission would be directly proportional to that of the reference mission. However, since there is model error, state determinations using the linearized equations would be expected to yield increasingly imprecise values as the separation range is increased. The resulting deviations between the true trajectory and the astronaut's estimate may cause an increase in both fuel and time over the previously mentioned values.

Model error due to eccentricity of the reference orbit was introduced by placing the target in an absolute orbit of finite eccentricity, while keeping its period and the initial relative state at the reference mission values. Since the linearized equations are only approximately valid in the case of eccentric orbits, it would be expected that state determination accuracy would be degraded and hence more fuel and/or time needed to rendezvous.

The main components of the measurement error, namely, sextant, window and operator error, were all lumped together. Errors were introduced by rounding sextant angles σ and δ to the nearest 2, 4, or 6 mr, giving rectangular density functions for the error, with standard deviations of 0.58, 1.15 and 1.73 mr respectively. With a finite number of readings per simulation and a finite number of runs, some anomalies can occur. For example, if the exact measurement was 10.2 mr, it was rounded to 10 mr at the lowest level and to 12 mr at the intermediate levels, giving errors of 0.2 and 1.8 mr respectively. Hence, instead of doubling the error as desired, the increase would be nine-fold in this case. The results must be interpreted with this in mind.

Thrusting may introduce control errors in burn direction and/or burn magnitude, which may be either random or biased. To simulate burn direction errors, the principal components of the desired burn were held fixed, while perpendicular components were introduced. The magnitude of the perpendicular component was made equal to the product of the principal component and the burn direction error in radians, while the sign of each was chosen at random on the reference mission of the test case and kept unchanged thereafter. Of course, the astronaut is unaware of the error at the time of the burn. In the case of burn magnitude error, the direction of the desired burn remained unchanged, but the magnitude was adjusted by the

product of the desired magnitude and the thrust error, expressed as a fraction. Since the intention was to simulate a fixed thruster bias, all burns of the test case were in error by the same proportion. Again the astronaut is unaware of the error and makes all subsequent calculations using his estimate of the burn magnitude rather than the true magnitude.

All of the perturbing influences discussed above were tested over a span which is larger than is expected in practice, simply to stress the rendezvous system heavily, and thereby determine if it breaks down under these conditions.

RESULTS

The results of the study for Case 1 are shown in Figs. 3–7. These figures display the ΔV and rendezvous time of each mission as a function of the magnitude of the individual error source. Since interest centers on the variations of ΔV or time relative to the reference mission, the results are normalized by dividing by the reference mission values. The amount by which the ΔV or time curve deviates from unity indicates the degree to which that particular error source imposes a penalty on the mission. A curve which remains essentially at unity over the test span means that the error source does not appreciably affect the operation of the rendezvous system. Minor deviations from unity can be due to errors in the simulation itself, finite rendezvous tolerance, etc., and are of no real consequence.

The first, and most significant, result is that rendezvous was achieved in all missions—no breakdowns occurred. Secondly, over the range of perturbing influences tested, the degradation in ΔV and time was not large for any of the error sources tested. Note that the ΔV curves have very nearly a unity ratio with their reference values on all five plots, Figs. 3–7. This desirable result reflects the solidity, or unperturbability, of the coelliptical approach trajectory. The circularization at a small differential altitude, as indicated in Fig. 2 at $t=3.12$ hr, gives the astronaut time to make a good state determination when the range is relatively short and the effect of errors is correspondingly diminished. The accumulated effect of prior errors, i.e. those made at long range, is readily removed without significant fuel or time penalty.

A statistically significant increase in ΔV occurred only in the eccentricity test, shown in Fig. 7, where values up to and including $\varepsilon=0.05$ were tried. An eccentricity of 0.05 in this context corresponds to carrying out rendezvous with a target in an orbit whose perigee is 75 nm and whose apogee is 445 nm above the earth's surface, a rather extreme orbit for a close-earth satellite. Yet even in this severe situation, the manual rendezvous system,

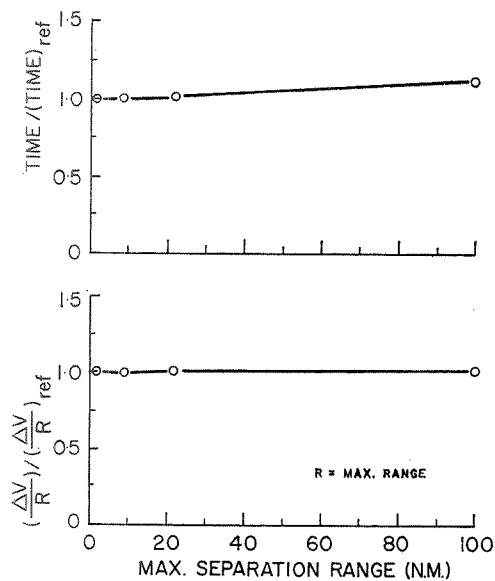


FIG. 3. Mission performance relative to separation range.

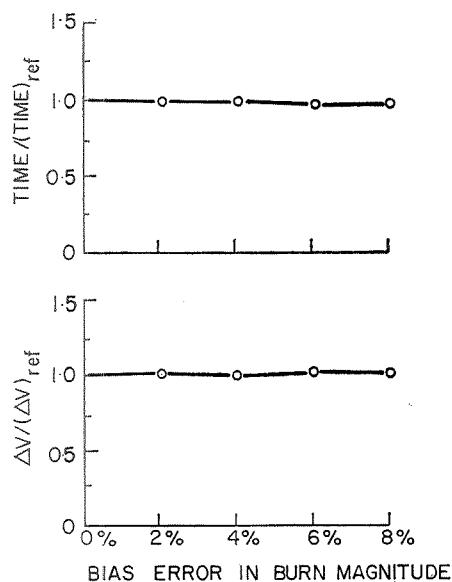


FIG. 4. Mission performance relative to bias in burn magnitude.

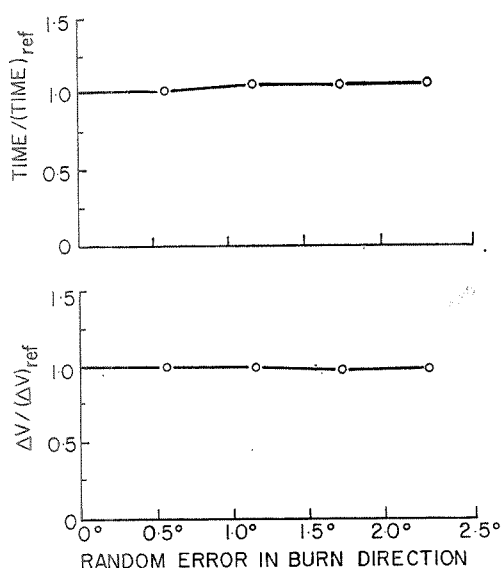


FIG. 5. Mission performance relative to error in burn direction.

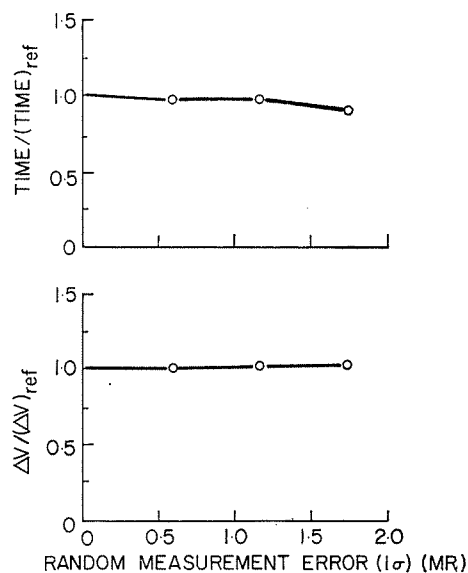


FIG. 6. Mission performance relative to measurement error.

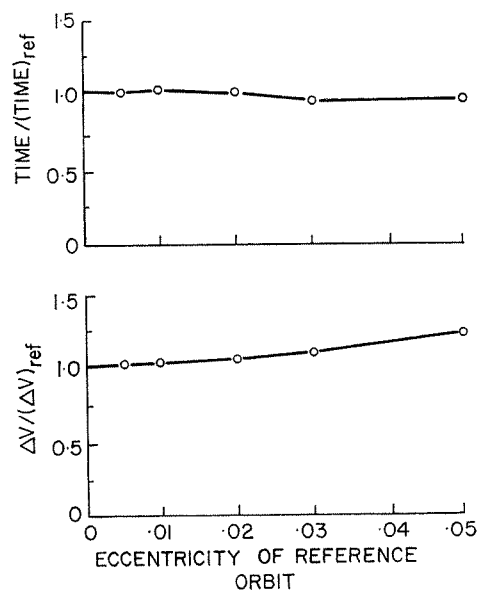


FIG. 7. Mission performance relative to reference orbit eccentricity.

using the constant-coefficient linear equations derived for circular orbits, performed satisfactorily. The ΔV penalty was 19 per cent at the highest value of eccentricity.

The resulting trajectory from one typical error run, the 1.73 mr measurement error mission, is shown in Fig. 8 to illustrate more fully the details of what happens when error sources are acting. In this figure, two trajectories are shown; the true value of the reference and the error trajectory, i.e. the trajectory resulting from the introduction of a perturbing source. Errors in the state following the measurement at $t=2.15$ result in a slightly

faulty flight plan, which, instead of being coelliptical, a straight line in these coordinates, wiggles up and down by a small amount. However, since neither apogee nor perigee were caused to change sign by the burns at $t=2.29$ and 3.07 , which set up this wiggling trajectory, no loss in ΔV -efficiency was incurred. A state-determination at 3.85 hr provides the data for the astronaut to make a mid-

ducted with the values of the major error sources set at levels which are believed to be representative of what would be expected in practice. For Case 1, the values used were: maximum range (nm)=21.2, $\varepsilon=0.015$, burn bias error=8 per cent, burn direction error (degrees)=1.15, measurement error (mr)=1.15, and k (knots/rad.)=0.0. The result of this particular mission is shown in Table 3 and Fig. 9.

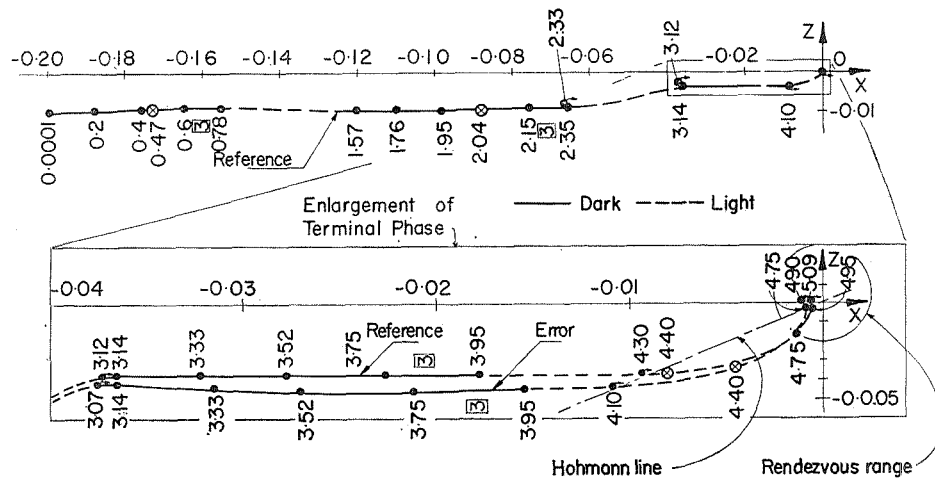


FIG. 8. Error trajectory—Case 1.
Measurement error=1.73 MR.

course corrective manoeuvre, specifically, a two-burn transfer to the target started upon reaching the Hohmann line and ended one-half orbit later at $t=4.90$. Rendezvous ΔV and time are very nearly the same as on the reference case.

Cases 2 and 3 were similarly studied and rendezvous was achieved in all missions. Although these cases represent vastly different initial conditions, penalty for errors were similar to the results presented for Case 1.

In actual flight, all error sources act simultaneously. Consequently, several missions were con-

TABLE 3. RESULTS OF MULTI-ERROR RUNS

	Reference	Error run
Total ΔV (knots)	6.2965	6.3247
Rendezvous time (hr)	5.09	6.16
$\Delta V/(\Delta V)_{\text{Ref.}}$	1.045	
Time/(Time) $_{\text{Ref.}}$	1.210	

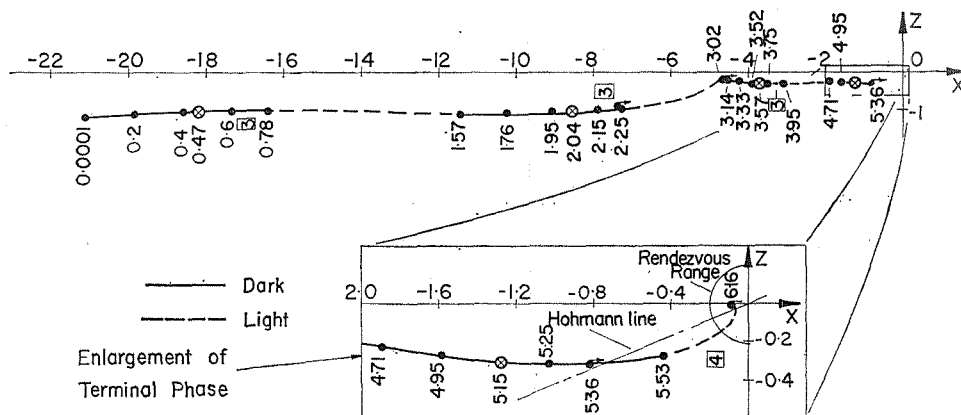


FIG. 9. Error trajectory—Case 1.
Multiple error sources.

Errors in state at 2.2 hr result in a wiggly instead of flat trajectory from 3.02 to 5.36 hr as indicated in Fig. 9. The average differential altitude of this part of the trajectory is smaller than on the reference mission, which was scaled, resulting in a slower approach to the target, and a later rendezvous. However, the wiggles do not significantly reduce fuel-optimality. A reasonably accurate state-determination at 3.85 hr suffices for the mid-course guidance, two-burn flight plan initiated at 5.36 hr, when the astronaut's estimated state crosses the Hohmann line. The second burn of this plan at 6.16 hr completes rendezvous in the light with a satisfactorily small miss. Actually, this miss would be accurately predicted by the state-determination made at 5.7 hr, which would enable the astronaut to come even closer by properly planned terminal maneuvers if he chose to do so. However, since his range at this time is 0.1 nm, this may not be warranted. The ΔV exceeded the reference ΔV by 4.5 per cent; the rendezvous time exceeded the reference value by 1.07 hr.

Similar results and conclusions were obtained from the other cases corresponding to a variety of choices for the error levels. These missions demonstrate the ability of the manual rendezvous system to function properly in the presence of realistic values of major error sources acting simultaneously. It should be noted that the effect of several simultaneous error sources is usually not as great as the sum of the effects taken one at a time—some statistical error cancellation occurs. Also, mid-course and terminal corrections remove much of the error which has accumulated up to that point. In this respect, the manual rendezvous system acts something like a sampled-data servo, periodically nulling its errors.

CONCLUSIONS

This study, in which a man and computer worked together in a conversational mode to simulate astronaut decision processes and their effect on the spacecraft orbit, demonstrates the feasibility of the manual rendezvous system from the standpoint of being able to work successfully in the presence of realistic sources of error. The system achieves rendezvous without undue fuel or time penalty for separation ranges as large as 100 nm, for reference orbit eccentricities up to 0.05, for burn magnitude bias up to 8 per cent, (with theory indicating that any constant burn bias is acceptable), with burn direction errors up to 2.29° , and with measurement errors up to 1.73 m standard deviation.

It is recommended that a study be made to determine methods for incorporating coelliptical flight plans for further improvement of the manual system.

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APPENDIX A

SUMMARY OF THE MANUAL RENDEZVOUS SYSTEM*

The significant space navigation and guidance problem of rendezvous can be solved by an astronaut without the aid of sophisticated instrumentation. In the manual rendezvous method [1], the astronaut bases all computations on measurements taken with a hand-held sextant, a clock and a collimated reticle pattern on the window of the spacecraft. A set of procedures has been developed which allows all data processing pertinent to the mission to be performed using charts, graphs, a slide rule and tables. Navigation is performed in a local tangent-to-the-flight-path coordinate system (LTCS) using linearized equations of motion to describe the relative motion of the astronaut's vehicle. The sextant measurements and a known thrust maneuver allow the astronaut to completely determine his relative position-velocity state in this frame. Guidance is performed by implementing one of three possible impulsive flight plans which minimize a linear combination of required fuel and mission time. Since errors and/or perturbations are usually present, the astronaut must continue the process by updating his state with new navigation data and adjusting his trajectory with mid-course and terminal guidance.

Linearized dynamical model

In order to simplify navigational computations, a model of the system dynamics was selected in which the spacecraft's equations of motion could be linearized about a nearby point in a reference orbit. The well-known linearized equations of

* The material is abstracted from Ref. [1].

motion for no-thrust trajectories may be written in the format:

$$\dot{X} = AX$$

whose solution for the state vector X at any non-dimensional time τ is:

$$X(\tau) = \Phi(\tau)X(0) \quad (A.1)$$

where

$$X \equiv \begin{bmatrix} x \\ y \\ z \\ x' \\ y' \\ z' \end{bmatrix} \quad A \equiv \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 0 & 0 \end{bmatrix}$$

$\tau \equiv \omega t$

$t \equiv \text{time}$

$$(\quad)' \equiv \frac{1}{\omega} \frac{d(\quad)}{dt}$$

$$\Phi(\tau) = \begin{bmatrix} 1 & 0 & 6(s-\tau) & 4s-3\tau & 0 & 2(c-1) \\ 0 & c & 0 & 0 & s & 0 \\ 0 & 0 & 4-3c & 2(1-c) & 0 & s \\ 0 & 0 & 6(c-1) & 4c-3 & 0 & -2s \\ 0 & -s & 0 & 0 & c & 0 \\ 0 & 0 & 3s & 2s & 0 & c \end{bmatrix}$$

$s \equiv \sin \tau \quad c \equiv \cos \tau$

Here x, y, z refer to the position vector components of the spacecraft relative to the reference point along the axes of a local tangent-to-the-flight path coordinate system (LTCS) in which the x -axis is forward in the reference orbital plane and tangent to the flight path, the y -axis is normal to this plane and in the direction of orbital angular velocity ω , the z -axis is mutually perpendicular to x and y in the right-handed sense, and the origin coincides with the reference point.*

Typical spacecraft trajectories resulting from the linearized model may be found in Ref. [1]. All trajectories may be classified according to their shape by the parameter, r , defined by

$$r = \frac{z_a}{z_b} \quad (A.2)$$

where z_a, z_b are the extreme values of z , z_a being the smaller in absolute value. Hence r ranges from -1 to $+1$. The precise location along the trajectory is determined by the parameter, $\bar{\tau}$, the dimensionless time interval between a particular point on a trajectory and the nearest time at which z reaches its maximum absolute value. In the absence of thrust, a vehicle will continue to traverse the same trajectory.

Measurement of line-of-sight orientation

In the LTCS, the ratios of the relative position variables are dependent only on the line-of-sight orientation and not the separation range. The line-of-sight orientation in the LTCS is described by the angles σ and δ defined by

$$\cot \delta \equiv \frac{x}{z} \quad \tan \sigma \equiv \frac{y}{(x^2 + z^2)^{\frac{1}{2}}} \quad (A.3)$$

State determination

From his measurement of σ and δ at various time instants, the astronaut would like to compute state. However, since any sequence of observed σ, δ pairs caused by some $X(t)$ could also have been caused by $KX(t)$ (in the linearized model), this computation is impossible unless a known velocity change at a known time is made, the change is observed, and the unknown K is calculated. For ease of astronaut computation, three specialized procedures have been developed for state determination. As shown in Table A.1, they differ in the sequence of measurements taken, the type of maneuver performed, and the computation algorithm used.

TABLE A.1. STATE DETERMINATION METHODS

Name	Method
3-instant out-of-plane burn	Three (σ, δ) pairs and thrust in y -direction between first and third measurements
4-instant in-plane burn	Four (σ, δ) pairs and thrust parallel to x, z plane between either third and fourth or first and second measurements
4-instant out-of-plane burn	Differs from above only in that the thrust is in the y -direction at time when $y=0$

Flight planning

Once the relative state of the vehicle has been determined by the navigation process, a suitable

* This system is identical with a local vertical coordinate system when the eccentricity of the reference orbit is zero. The desirable properties of LTCS in the case of an eccentric reference orbit are discussed in Ref. [5].

flight plan must be developed. A flight plan specifies the number, magnitude, time and direction of the thrust impulses.

Since the out-of-plane and in-plane motion in the linearized equations are decoupled, the astronaut need only consider the in-plane rendezvous problem once the correct plane change has been effected by an out-of-plane impulse applied at any convenient crossing of the target orbital plane.

Three types of in-plane flight plans, shown in Table A.2, are used in the manual scheme. They were developed to satisfy a Fuel-Time Optimality Criterion while still being relatively simple to implement. Optimality is defined as minimization of a rendezvous cost equal to a linear combination of the fuel and transfer time of the mission.

$$\text{Cost} = \Delta V + k\tau.$$

Here ΔV is a measure of the expended fuel and is equal to the sum of the magnitudes of the velocity changes caused by thrust impulses. The constant k is a weighting coefficient and τ is the nondimensional transfer time. Note that in one reference orbit period τ changes by 2π . The weighting constant k is a measure of the relative importance of fuel and transfer time. In comparing two possible flight plans, the astronaut may elect to spend up to k additional units of ΔV to save a unit of time or *vice versa*.

TABLE A.2. IN-PLANE FLIGHT PLANS

Name	Method
Three-burn plan	Thrust impulses are directed along the local horizontal and separated in time by half-integer multiples of the reference orbit. The burns are always made at apogee or perigee of the relative trajectory
Wait-and-two-burn plan	A minimum fuel, two-horizontal impulse orbit transfer in which the time between impulses is unrestricted
Immediate two-burn plan	A terminal phase, non-horizontal-impulse orbit transfer of comparatively short duration used when the fuel cost of the remaining portion of the mission is less important than the transfer time

In use, the astronaut selects a value of k . He consults a set of Flight Planning Charts which tell him, for any state, what kind of flight plan to use.

Résumé—Il a été développé un système de navigation de guidage et de commande d'un vaisseau spatial vers un rendez-vous avec un but orbital, entièrement basé sur des

observations au moyen d'instruments manipulés à la main et dépourvus d'énergie auxiliaire et sur des calculs entièrement effectués manuellement. Le présent article décrit les résultats d'une simulation de ce système avec interaction au moyen d'une série choisie de missions de rendez-vous, qui montrent la possibilité de les réaliser en présence d'erreurs réalistes. La simulation est effectuée par un opérateur expérimenté, "l'astronaute", travaillant au moyen d'une conversation avec un calculateur numérique de la dimension d'une table. Le calculateur représente l'ambiance et l'opérateur représente le procédé de prise de décision par l'astronaute. Des enregistrements sont donnés pour montrer l'augmentation de la consommation de carburant et de la durée du rendez-vous en fonction de l'importance des principales sources d'erreurs, notamment: de l'erreur de simulation provoquée par la distance de séparation, de l'erreur de simulation occasionnée par l'excentricité de l'orbite de référence, des erreurs de commande en direction et en grandeur, des vecteurs de combustion et de l'erreur de mesure. Il est montré que le système manuel de rendez-vous est fortement convergent, de sorte que la véritable pénalisation des erreurs n'est pas constituée par l'échec du rendez-vous mais plutôt par l'accroissement de la consommation de carburant et/ou de la durée qui auraient été autrement nécessaires.

Zusammenfassung—Ein System zur Navigation, Führung und Regelung eines Raumschiffes für ein Rendezvous mit einem Ziel auf einer Kreisbahn, das ganz auf Beobachtungen mit Handbetätigung, mit leistungsarmen Instrumenten und lediglich manuell ausgeführten Rechnungen basiert, wurde entwickelt. Beschrieben werden die Ergebnisse der Simulation bei der Zusammenwirkung dieses Systems mit einer ausgewählten Menge von Rendezvous-Aufträgen, die die Durchführbarkeit bei Gegenwart realer Fehler zeigen. Die Simulation wird durch einen trainierten Operateur, den "Astronauten", durchgeführt, der mit einem Pultdigitalrechner als Partner arbeitet. Der Rechner repräsentiert die Umgebung und der Operator repräsentiert den Entscheidungsprozeß des Astronauten. Kurven werden angegeben, die das Anwachsen des Brennstoffverbrauches und der Rendezvous-Zeit als Funktion der Größe der hauptsächlichsten Fehlerquellen/zeigen, nämlich Modellfehler auf Grund des trennenden Abstands, Modellfehler auf Grund der Exzentrizität der Bezugsbahn, Regelabweichungen der Bremsvektoren nach Richtung und Größe und schließlich Meßfehler. Gezeigt wird, daß das manuelle Rendezvous-System schnell konvergiert, so daß die wirkliche Erschwerung durch Fehler nicht so sehr in Bezug auf das Rendezvous, sondern mehr dabei liegt, ob mehr Brennstoff und/oder Zeit benötigt wird als sonst.

Резюме—Была разработана система навигации, направления и управления космическим кораблем к встрече с целью на орбите, полностью основанная на наблюдениях при помощи аппаратов используемых вручную, без всякой вспомогательной энергии, и на вычислениях также полностью выполняемых вручную. Настоящая статья описывает результаты моделирования этой системы с взаимодействием при помощи ряда выбранных заданий встреч, каковые показывают возможность их осуществления в присутствии реальных ошибок. Моделирование выполняется опытным оператором, "космонавтом", работающим с помощью разговора с цифровой вычислительной машиной столбного размера. Вычислительная машина представляет собой окружение а оператор представляет собой процесс принятия решения космонавтом. Даются записи для показания увеличения расхода горючего и длительности встречи в зависимости от важности главных источников ошибок в частности: ошибки моделирования вызванной эксцентricностью орбиты референции, ошибок управления по направлению и по величине векторов сгорания и ошибки измерения. Показывается что ручная система встречи обладает высокой сходимостью, так что действительный штраф за ошибки не состоит из неуспеха встречи а скорее из повышения расхода горючего и/или длительности которые были-бы иначе необходимы.

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INFLUENCE OF PERTURBING EFFECTS
ON A MANUAL RENDEZVOUS SYSTEM

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ABSTRACT

A system for navigation, guidance, and control of a spacecraft to rendezvous with an orbiting target, based entirely on observations by handheld, unpowered instruments, and computations done entirely by hand, has been developed. This paper describes results of an interactive digital simulation of this system through a selected set of rendezvous missions. A previously reported study to evaluate the influence of error sources on the system is extended to two new test cases. In addition, several perturbing influences not covered heretofore are examined, specifically: errors in the method of star sight averaging made to compensate for non-simultaneity of a pair of sightings, incorrect knowledge of the interceptor spacecraft's orbital period, and astronaut computation error. An activity chart is included which shows the apportionment of two astronauts' time in carrying out rendezvous using the manual system. It is shown that rendezvous is achieved on all error missions without undue increase in fuel and/or time relative to an "error-free" mission.

INTRODUCTION

An interactive man/computer simulation designed to evaluate the manual rendezvous system [1] is described in Reference 2.* In that paper, the effect of several major error sources on one particular test case (Case 1) was examined through a selected set of rendezvous missions. The error sources studied were: mathematical model error due to finite separation range between interceptor and target; model error due to eccentricity of the reference orbit; control errors in direction and magnitude of burn vectors, and measurement error. In this paper, these error sources are applied to two additional test cases (Cases 2 and 3). Furthermore, the following new sources of error are studied: errors in the method of star sight averaging made to compensate for non-simultaneous sightings; incorrect knowledge of the interceptor's orbital period, and astronaut computation error.

*Appendix I summarizes the essential features of the simulation.

One question which arises in the application of the manual rendezvous system concerns the length of time required on the astronaut's part to carry out all of the required functions, including measurements, computations, and maneuvering. This problem was studied and results in the form of an astronaut activity chart are presented below.

DESCRIPTION OF TEST CASES

A "test case" is described by specifying the reference orbit (the orbit of one of the vehicles), the relative position and velocity vectors between the interceptor and target at the start of the rendezvous phase ($t = 0$ hrs), and the weighting constant k which the astronaut assigns as a measure of the priority between fuel and time.* Table I lists the initial conditions which establish Test Cases 2 and 3.

TABLE I
TEST CASE INITIAL CONDITIONS

	<u>Case 2</u>	<u>Case 3</u>	<u>Units</u>
x_0	.2	.126	nm
y_0	.028	.085	nm
z_0	.02	-.126	nm
$\dot{x}_0$	-.144	.256	knots
$\dot{y}_0$	.08	-.1248	knots
$\dot{z}_0$	.16	.382	knots
ϵ	.0	.0	
T	1.5697	1.5697	hrs
PC	.35T	.19T	
DB	.0	.2	hrs
k	.0102	.0716	knots/rad

*Mission cost = $\Delta V + k\omega t_f$, where ΔV is the fuel used in units of characteristic velocity, ω is the mean motion of the reference orbit, and t_f is the time to rendezvous measured from $t = 0$ hrs.

The symbols used in all tables are defined in Nomenclature. For reasons to be discussed in the next section, the relative state vectors have been normalized for an initial separation range of 0.2 nautical miles (nm). Figure 1 shows the true relative trajectory for each case in the absence of thrusting.

In this paper, x, y, z will refer to the position vector components of the spacecraft relative to a reference point along the axes of a local tangent-to-the-flight path coordinate system (LTCS) in which the x -axis is forward in the reference orbital plane and tangent to the flight path, the y -axis is normal to this plane and in the direction of orbital angular velocity ω , the z -axis is mutually perpendicular to x and y in the right-handed sense, and the origin coincides with the reference point.

STRATEGIES FOR THE TEST CASES

The criteria of Table II define an "error free" or "reference" mission on the digital simulator. The data given in Table I establishes initial conditions for the reference mission of Cases 2 and 3.

The test plan calls for evaluating the error sources one by one. A given error-producing effect is introduced into a rendezvous mission that has all other error sources held at the reference mission values, and a new rendezvous mission is run with this error source. A series of such runs is made, with the error source set successively at higher values. The successive error missions will be denoted by the magnitude of the individual error source present-- e. g. the 10-mile range mission of Case 2, the .03 eccentricity mission of Case 3, etc.

To achieve rendezvous from a given set of initial conditions* requires the implementation of strategy -- the particular sequence of measurements and the kind(s) of flight plans used by the astronaut during a mission. In order to evaluate the influence of each individual error source, a common strategy was normally used for all missions of a given test case regardless of the magnitude or type of error present. The use of one common strategy makes it possible to compare the error sources to determine their relative influence on the manual rendezvous scheme; this comparison would be less meaningful if different strategies were used for successive missions.

*The operator achieves rendezvous with the target when both:

Final Separation Range	$\leq .01 \times (\text{Maximum Separation Range})$
Final Velocity Magnitude (knots)	$\leq 4 \times (\text{numerical value of Final Separation Range in nm})$

TABLE II
STANDARD CONDITIONS FOR
"REFERENCE" MISSIONS

<u>Error Source</u>	<u>Magnitude</u>
Separation Range	.02 nm at initial time -- this short range effectively nulls the model error in the linearized equations due to finite range
Eccentricity of Reference Orbit	The target orbit is circular ($\epsilon = 0$) with a radius of 3700 nm -- this nulls the model error in the linearized equations due to eccentricity.
Control Errors in Burn Vector	$\left\{ \begin{array}{l} \text{Magnitude -- accurate to nearest } 5 \times 10^{-9} \text{ knot*} \\ \text{Direction -- accurate to nearest } 5 \times 10^{-9} \text{ rad*} \end{array} \right.$
Measurement Error	Line-of-sight angles σ and δ -- accurate to nearest 5×10^{-9} rad*
Error in Knowledge of Interceptor's Orbital Period	The exact value of the reference orbital period 1.5697 hrs must be used.
Star Sight Averaging Error	Time between measurements (Δt) used in the averaging process = 0.0 min
*These values establish the level of round-off error in entering the data.	

Case 2

Table III presents the astronaut's reasoning and the resulting strategy for Case 2. Figure 2 shows the in-plane (x-z) relative trajectory in LTCS when the Case 2 strategy is used on the reference mission.

After determining state during the first dark period, the astronaut consults his flight-planning chart [1] and finds that a three-burn maneuver is needed to achieve rendezvous. The burns are to be separated by 1-1/2 and 1/2 reference orbit period respectively.

After executing the first burn of the maneuver, which lowers apogee, he re-determines state during the next dark period and updates his guidance with a new three-burn plan. The first burn is designed to correct errors imparted by the original in-plane burn; the

second and third burns will be very similar in magnitude, direction, and time to those of the original plan. Specifically, the second burn will raise the interceptor's perigee to coincide with that of the target's orbit, and the third burn will bring the vehicle to rest in a rendezvoused position. Note that for missions in which an error source is present, the transfer orbit may not meet the target at the scheduled time of the third burn due to deviations between the true trajectory and the astronaut's estimate. If this occurs, it is necessary to replace the final burn with an immediate two-burn maneuver as called for by the flight-planning charts.

TABLE III
STRATEGY - CASE 2

	<u>Event</u>	<u>Time</u> (hrs)	<u>Reasoning</u>
1.	DB	.0	Dark begins.
2.	●	.0 .2 .4	These measurements will determine the times of interceptor orbit perigee-apogee passage and the orbital plane-crossing time.
3.	⊗	PC	Calibrated thrust removing approximately 1/2 remaining out-of-plane velocity magnitude. Residual velocity is purposely left in to aid in future state determinations.
4.	●	.65	A measurement after the out-of-plane thrust is made to set up a state determination using 3 .
5.	3		Measurements at .2, .4, .65 hrs and out-of-plane thrust are used to determine the state at .4 hrs.
6.	SP		State is projected to the next perigee passage time. The astronaut decides to initiate a three-burn rendezvous maneuver.

TABLE III (continued)

7.	3B		Calculations of the components of each burn are made. The first burn lowers apogee, the second burn (1-1/2 period later) raises perigee, and the third burn (1/2 period after the second burn) will complete rendezvous if the influence of errors is not too large.
8.	●	.78	Measurement taken just before leaving the dark. It is good practice to take extra measurements in case a previous measurement should turn out to be faulty.
9.	DE	.7848	Dark ends.
10.	☛	P _e	First burn of maneuver.
11.	SP	1.0 1.3	The known state at .4 hrs is projected forward to monitor progress during the light period.
12.	DB	1.5698	
13.	●	1.57 1.8 1.95	See Event 2.
14.	⊗	PC	See Event 3.
15.	●	2.25	See Event 4.
16.	3		Measurements at 1.8, 1.95, and 2.25 hrs and out-of-plane thrust allow the astronaut to re-determine state at 1.95 hrs.
17.	SP		State is projected to the next perigee passage time. Using his new state made at closer range, the astronaut decides to up-date his original guidance by implementing a new three-burn plan with 1/2 period between each of the burns.

TABLE III (continued)

18.	3B		Calculations of the burn components for the new flight plan are made.
19.	●	2.35	See Event 8.
20.	DE	2.3546	
21.	●→	P_e	First-burn of the new three burn maneuver. This burn is usually a fine adjustment of the original in-plane burn made one orbit earlier.
22.	SP	2.6 2.9	See Event 11. (known state at 1.95 hrs.)
23.	DB	3.1394	
24.	●	3.14	Extra measurement upon entering the dark.
25.	●→	A_p	Second-burn of maneuver designed to lift interceptor's perigee to coincide with that of target's orbit.
26.	●	3.4 3.55	The first two measurements needed for 3 .
27.	⊗	PC	Since this will be the last scheduled state determination, all of the remaining out-of-plane velocity is removed.
28.	●	3.85	Third measurement needed for 3 .
29.	3		Measurements at 3.4, 3.55, 3.85 hrs and out-of-plane thrust are used to determine state at 3.4 hrs.
30.	SP		The astronaut projects his state to the time of the third burn and observes if its execution will allow him to rendezvous. If not, he can calculate the burn components for an immediate two-burn maneuver beginning at 3.95 hrs and use this for his terminal guidance.

TABLE III (continued)

31.	●	3.92	See Event 8.
32.	→●	P_e	If the astronaut decides his trajectory is on course, the final burn of the three-burn maneuver is made. He is now in a favorable position for docking.

Case 3

Figure 3 shows the in-plane relative trajectory in LTCS when the Case 3 strategy as described in Table IV is used on the reference mission.

This randomly chosen test case presents the astronaut with among the most difficult set of initial conditions from which to achieve rendezvous. The astronaut's vehicle is below and forward of the target, moving away and causing the separation range to markedly increase in a short period of time. Under such conditions, rapid state determination and maneuvering is essential so that the target will not be lost. However, although the astronaut may be aware of his predicament by observing the receding target, he cannot take readings for .2 hrs until the dark period begins. Ordinarily, for fuel-optimality, one chooses to wait to make an out-of-plane calibrated thrust maneuver at a plane-crossing. In this case, because of the urgency for determining state quickly, fuel-optimality is sacrificed, and an out-of-plane burn is made at the earliest time consistent with the 3 procedure, which the astronaut then uses to determine state. The first two burns of a three-burn flight plan establish him at a higher average altitude than the target, so that the range closes. From this condition, he elects to coast until the separation range is reduced and state is accurately redetermined. During a two-orbit coasting period, the astronaut re-determines state and verifies that the first two burns of the maneuver have placed him on the proper transfer trajectory for rendezvous. If he is off course, as may be the situation for an error mission, terminal guidance consists of a three-burn maneuver beginning one orbit before the time of the originally scheduled final burn.

TABLE IV
STRATEGY - CASE 3

	<u>Event</u>	<u>Time</u>	<u>Reasoning</u>
1	DB	.2	
2	●	.2	First measurement for use in 3 procedure.
3	⊗	.3	Astronaut chooses a time instant and makes a calibrated thrust of approximately 1/2 his remaining out-of-plane velocity. Had normal procedures been followed, a calculated plane-crossing time of 1.08 hrs would have resulted.
4	●	.4 .6	Second and third measurements needed for using 3 . The times of apogee and perigee have now been calculated.
5	3		Measurements at .2, .4, .6 and estimated thrust are used to determine state at .6 hrs. Note: 3 can be used even though the burn is between measurement 1 and 2 instead of 2 and 3 as in the other test case.
6	SP		The known state is projected to the perigee passage time and the astronaut quickly decides to use a three-burn maneuver.
7	3B		Calculation of the components of each burn. The first two burns raise the average altitude of the interceptor higher than the target so that the separation range will close.
8	●→	P _e	First burn of maneuver.
9	●	.98	See Event 8 - Case 2.
10	DE	.9848	

TABLE IV (continued)

11	SP	1.00 1.35 1.6	The known state at .6 hrs is projected forward to monitor progress during the light period.
12		A _p	Second burn of maneuver at 1/2 reference orbit after first burn.
13	DB	1.7696	
14		1.77	See Event 2 - Case 3.
15.		PC	A calibrated thrust is made at the plane-crossing time determined from the previous state determination. One-half of the remaining out-of-plane velocity is removed, the residual being left for future state determinations.
16		2.0 2.3	See Event 4 - Case 3.
17	3		Measurements at 1.77, 2.0, 2.3 and calibrated thrust used to determine state at 2.3 hrs. The astronaut sees he is satisfactorily closing on the target and decides to monitor his progress for at least one more orbit before applying midcourse corrections.
18		2.55	See Event 8 - Case 2.
19	DE	2.5544	
20	SP	2.9 3.15	See Event 11 - Case 3 (known state at 2.3 hrs).
21	DB	3.3392	
22-25			The events during this phase of the dark period are identical to Events 14-17. Measurements are taken at 3.34, 3.6, and 3.85 hrs with state re-determined at the latter time.

TABLE IV (continued)

26	SP		The astronaut decides if any corrective maneuvers are necessary by determining where he will be in one more orbit, the scheduled time of the final burn. If rendezvous is not going to be achieved, a three-burn corrective maneuver is initiated beginning at approximately 4.0 hrs. The first two burns will correct any errors in his trajectory so that the third burn can bring him to rest in a favorable position for docking.
27(a)		P_e	First burn of possible corrective maneuver.
27(b)		4.1	See event 8 - Case 2.
28	DE	4.1240	
29(a)	SP	4.35 4.6	See event 11 - Case 3 (known state at 3.85 hrs)
29(b)		A_p	Second burn of possible corrective maneuver
30	DB	4.9088	
31		4.91	See event 2 - Case 3.
32		PC	All of the remaining out-of-plane velocity is removed with this burn.
33		5.15 5.4	See event 4 - Case 3
34	3		Measurements at 4.91, 5.15, and 5.4 hrs are used to determine state at 4.91 hrs.
35	SP		State is projected to the time of the final burn (either the third burn of the original flight plan or the corrective plan) and the astronaut verifies he will rendezvous.

TABLE IV (continued)

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After completing the final in-plane burn, the astronaut is in position for docking.

PROCEDURE

With the magnitudes of the error sources set at the "error-free" conditions of Table II, the complete reference mission for each test case was simulated and data was obtained on both the total fuel expenditure, as measured by ΔV , and time in achieving rendezvous. This data, together with cost, appears in Table V. Unless otherwise specified, ΔV is expressed in knots, range in nm, and time in hours (hrs) throughout.

TABLE V
RESULTS OF REFERENCE MISSIONS

	<u>Case 2</u>	<u>Case 3</u>
Total ΔV (knots)	.1630	1.6013
Rendezvous Time (hrs)	4.01	5.59
Cost (knots)	.3266	3.2026

Then, a given error-producing effect was introduced, each test case re-run with all other error sources held at the reference mission values, and new results on fuel and time obtained. A series of such rendezvous missions was simulated with the magnitude of the error source of interest set at successively higher values. Any velocity remaining when the rendezvous criterion is met is added, using the "cost of a trajectory" formula [1], to the ΔV spent in burns, to arrive at the total ΔV for the mission. The excess ΔV or cost required in a case with error present is an indication of the mission penalty which such an error imposes.

The equations of motion used in the manual rendezvous system are derived from first-order linear perturbations around a circular reference orbit; hence finite separation range and reference orbit eccentricity introduce a mathematical model error which was tested at successively higher levels in the series of range and eccentricity error missions, respectively.

Burn direction errors were simulated and their effects determined by introducing small perpendicular components of incremental velocity gained by thrust in each burn maneuver, the sign of the component selected randomly for each maneuver of the reference case. Burn magnitude errors were treated as a fixed bias for all burns of each error mission.

To simulate measurement error, including sextant, window, and operator contributions, fixed angle errors were applied to both in-plane angle δ and out-of-plane angle σ , with the sign randomly selected for each measurement time on the reference case.

In the manual method, two sextant sightings are taken at each "measurement time," that is, the target-to-star angle is read on the sextant for each of two different stars. A simple averaging procedure is intended to yield a close approximation of the values which would have been obtained if the two readings were taken simultaneously. The procedure used in the simulation was to take readings of δ and σ at $(t_1 - \Delta t/2)$ and at $(t_1 + \Delta t/2)$, and average them to obtain the approximate values at t_1 . Here t_1 is the desired measurement time, and Δt is the time between the first and last measurement. This procedure correctly simulates what would be obtained in actual flight under the same circumstances [3].

The manual rendezvous system requires knowledge of the orbital period of one of the vehicles. Error in knowledge of the reference orbit period T was simulated by introducing a constant error in the period wherever it was needed in the astronaut's computations.

All of the perturbing influences discussed above were tested over a span which is larger than expected in practice, simply to stress the rendezvous system heavily, and thereby determine if it breaks down under these conditions.

RESULTS

The results of the study for Cases 2 and 3 are shown in Figures 4-10. These figures display the ΔV and cost of each mission as a function of the magnitude of the individual error source. Since interest centers on the variations of ΔV or cost relative to the reference mission, the results are normalized by dividing by the reference mission values. The amount by which the ΔV or cost curve deviates from unity indicates the degree to which that particular error source imposes a penalty on the mission. A curve which remains

essentially at unity over the test span means that the error source does not appreciably affect the operation of the rendezvous system. Minor deviations from unity can be due to errors in the simulation itself, finite rendezvous tolerance, etc., and are of no real consequence.

Continuing the trends found for Case 1 [2], the first, and most significant, result is that rendezvous was achieved in all missions -- no breakdowns occurred. Secondly, over the range of perturbing influences tested, the degradation in ΔV and cost was not overly large for any of the error sources tested. Closer examination of Figures 4-10, and the results for Case 1, leads to the conclusion that the error sources fall into two groups: those which have a negligible influence on the trajectory (at least to the maximum value of the error source studied and reported here), and those which do impose some performance penalty roughly proportional to the magnitude of the error source. In the former category are: separation range burn magnitude, knowledge of orbital period, and star sighting averaging error. In the latter category are reference orbit eccentricity, measurement error, and burn direction error.

The resulting trajectory from three typical error runs are shown in Figures 11-13 to illustrate more fully the details of what happens when an error source is acting. In each of these figures, two trajectories are shown; the true value of the reference trajectory and the "error" trajectory, i. e., the trajectory resulting from the introduction of a perturbing source. Figure 11 shows the effects of .02 eccentricity of the reference orbit on Case 2. Even before any in-plane maneuvers, the error trajectory deviates from the reference trajectory in the x-component. This is the characteristic appearance of the model error induced by eccentric orbits, a phenomenon more fully explored in Reference 4. Despite the eccentricity error, there is no difficulty encountered in this case in applying the standard procedures; rendezvous is obtained using the same strategy as on the reference mission (Table III), with a small fuel penalty (6.8%).

Effects of error in burn direction for Case 2 are illustrated in Figure 12. The first state-determination at $t=.72$ has errors due to the components of $\dot{x}$ and $\dot{z}$ introduced unknowingly when the first out-of-plane burn was made at $t=.55$. These state errors, particularly the error in $\dot{x}$, propagate to large position deviations from the reference trajectory by $t=2.35$. An error in the determination of $\dot{x}$ made early in the trajectory would be prevented from further propagation if the astronaut could make a perfect state-determination at $t=2.3$. But, by virtue

of the burn angle error, the burn at $t = 2.11$ also has $\dot{x}$ and $\dot{z}$ components which are unknown to the astronaut, and which therefore introduce a fresh $\dot{x}$ and $\dot{z}$ error in the state determined at $t = 2.3$. Even more $\dot{z}$ error is introduced by the erroneous burn direction at $t = 2.45$, resulting in the miss of the Hohmann line (the line in the relative coordinate system along which the first burn of a two-burn, circle-to-circle, rendezvous using a Hohmann transfer would occur). State at $t = 3.9$ reveals that the current flight plan will not rendezvous, and hence the astronaut immediately implements a correcting two-burn maneuver which results in a satisfactory rendezvous, but at the price of increased fuel and time. A similar two-burn corrective maneuver is needed on the higher levels of the measurement error missions of Case 2 as well.

Figure 13 pertains to eccentricity = .03 on Case 3. Here, errors in the first two burns ($t = .88$ and $t = 1.68$) made on the basis of an inaccurate early state determination at $t = .75$ result in too high a relative apogee and perigee, with a resulting overly rapid approach to the target. Two new state determinations at $t = 2.35$ and $t = 3.9$ reveal the overshoot to be expected if the original flight plan is continued. Corrective action at $t = 4.0$ leads to a successful three-burn flight plan. This particular mission exhibits as marked a deviation from the reference trajectory as one finds in any of the error missions of Case 3, except for the .05 eccentricity case. Some of the error trajectories of Case 3 did not need a three-burn corrective maneuver like the one shown in Figure 13. While none of the Case 3 error trajectories required the more expensive two-burn corrective maneuvers, this seeming advantage may in fact only be a reflection of the larger rendezvous tolerance assigned to Case 3.

In actual flight, all error sources act simultaneously. Two runs were made to simulate this condition, one for each of the test cases. Error sources were set to the values of Table VI, which are believed to be representative of values expected in practice. The results of these runs are shown in Table VII and in Figures 14 and 15.

The reference ΔV for a given case was calculated by multiplying the ΔV of the corresponding reference mission by the same scale factor as that used to establish initial separation range. The reference rendezvous time is the same as that on the reference mission. The value of k was determined by dividing the reference ΔV by the reference time in appropriate units. The k -value, the actual ΔV , and the actual rendezvous time were used in the standard formula to compute the cost of the error mission.

TABLE VI
VALUES OF PERTURBING INPUTS
AND ERROR SOURCES FOR MULTI-ERROR RUNS

	<u>Case 2</u>	<u>Case 3</u>
Maximum Range (nm)	20.0	34.0
Eccentricity	.015	.015
Burn Bias Error (%)	4	4
Burn Direction Error (degrees)	1.15	1.15
Measurement Error (mr)	1.15	1.15
k (knots/rad)	1.02	.716

TABLE VII
RESULTS OF MULTI-ERROR RUNS

	<u>Case 2</u>	<u>Case 3</u>
Total ΔV (knots)	20.2175	17.0574
Rendezvous Time (hrs)	4.30	5.57
Cost (knots)	37.615	33.01
Reference ΔV (knots)	16.30	16.013
Reference Rendezvous Time (hrs)	4.01	5.59
Reference Cost (knots)	32.66	32.026
$\Delta V / (\Delta V)_{\text{Ref.}}$	1.240	1.065
Cost / (Cost) _{Ref.}	1.156	1.031

A Case 2 error trajectory for multiple error sources appears in Figure 14. As in Figure 12, the accumulated effect of errors is corrected by a two-burn maneuver at short range ($t = 3.95$ to $t = 4.30$). A

Case 3 error trajectory for multiple sources is shown in Figure 15. As in Figure 13, a final three-burn flight plan initiated at $t = 4.0$ successfully corrects the results of previous errors.

The multiple-error missions of Cases 2 and 3 (Figures 14 and 15) as well as the multiple-error mission of Case 1 [2], demonstrate the ability of the manual rendezvous system to function properly in the presence of realistic values of major error sources acting simultaneously. It should be noted that the effect of several simultaneous error sources is usually not as great as the sum of the effects taken one at a time -- some statistical error cancellation occurs. Also, midcourse and terminal corrections remove much of the error which has accumulated up to that point. In this respect, the manual rendezvous system acts something like a sampled-data servo, periodically nulling its errors.

STUDY OF THE MANUAL COMPUTATION PROCESS

Since the manual method is based on the astronaut performing all computations himself, with the help of special computational aids, it is of interest to know how long these computations take, and how accurately they can be made.

A rudimentary study of the manual computation process was made, and the results, which must be regarded as preliminary, are presented in Table VIII. The method was to perform a complete state determination and selection of a flight plan for two different sets of initial conditions, using only the information available to the astronaut in the real situation, i. e. sextant readings, time, and knowledge of the period of one of the two vehicles. The initial conditions were selected, for convenience, to be the same as the initial conditions for Cases 1 and 3, for which "sextant readings" were already available. In each case, state was determined using the applicable procedures defined in Reference 5 (Appendix XVII), but unlike the prior discussions, all computations were done by hand.

The person who performed this study was a senior student in the Department who had approximately ten hours of training in the system (approximately 80 hours is felt to be necessary to train an astronaut). It is anticipated that a fully trained user could improve in proficiency so that the computation times would be reduced below the figures given in Table VIII, adjusted for 50% learning improvement.

The following computational aids were used: a slide-rule with a ruler scale, protractor, dividers, phase-plane plotter, graph paper, Universal Trajectory Charts, ABC Tables, r - τ Tables, and Flight Planning Charts [5]. Manual computation times for the various steps

are given, with the total time shown for one pass through the navigation and guidance equations. The steps involved would typically have to be performed more than once in any given rendezvous mission. For the mission of Case 3, an Astronaut Time-Activity Chart (Figure 16) has been developed, closely following the events of Table IV. The time allotment for each activity was that given in Table VIII adjusted for 50% learning improvement.

The time required to do two important sets of calculations, the reduction of sighting angles from celestial to LTCS coordinates and the determination of windshield X-Y values for thrust alignment, were not experimentally determined in this study. The computation time for the former would depend on which of the three possible methods suggested in Reference 5 is implemented; the shortest time would be obtained using the mechanical spherical triangle solver [6]. These computations were included in the activity chart, however, using assumed values of 6 minutes and 1.5 minutes, respectively, which are felt to be reasonable.

TABLE VIII
TIME REQUIRED FOR MANUAL COMPUTATIONS

<u>Designation</u>		<u>Case 1</u> (min)	<u>Case 3</u> (min)
IPNS	Normalized State Determination (r , $\bar{r}$ found)	10	11
SP ₁	Project Normalized State and In-plane Range at Two Times	6	6
PC	Determination of Plane-Crossing Time	4	-
SP ₂	Same as SP ₁	4	5
3	Dimensional State Determined Using 3-Instant Out-of-Plane Procedure	9	11
SP ₃	Projection of Dimensionalized State	5	5
3B	Determine Burn Magnitudes for Three-Burn Flight Plan	<u>4</u>	<u>5</u>
TOTAL		42	43

An indication of the accuracy of the manual computations is obtained by noting the differences between the hand-computed and digital computer results of Table IX. It is believed, but not verified, that this level of accuracy will not degrade mission performance to a significant degree.

TABLE IX
COMPARISON OF HAND-CALCULATED VALUES
WITH COMPUTER VALUES

Symbol	<u>Case 1</u>		<u>Case 3</u>	
	Hand	Computer	Hand	Computer
ξ	-1.48	-1.50	-1.59	-1.62
η	- .0611	- .0530	.513	.482
r	.880	.899	.190	.168
$\bar{r}$	1.70	1.61	1.21	1.11
PC_1	.471	.471	1.86	1.87
PC_2	1.25	1.26		
A_p or P_e	.760	.782	.902	.876
PLAN	3-Burn	3-Burn	3-Burn	3-Burn

CONCLUSIONS

This study further substantiates the conclusions put forth in Reference 2, namely, that the manual rendezvous system can successfully accomplish its mission in the presence of realistic sources of error.

Based on the reasonable expectation that an astronaut can be trained to perform the calculations required by the manual system in one-half the times shown in Table VIII, the system is capable of being used in real time.

Over the range of values tested in this simulation, the following error sources were found to have the greatest effect: reference orbit eccentricity, burn direction-error, and measurement error. Bias error in burn magnitude, incorrect knowledge of interceptor period, and

effect of non-simultaneous measurements proved to be relatively unimportant from a total mission standpoint.

It is recommended that the following areas be investigated in any continuation of the manual rendezvous program: 1) inclusion of computation error in the simulation, 2) errors in knowledge of plane of reference orbit, 3) error in knowledge of position in the reference orbit, and 4) re-examining the basic computational methods to see if they can be made simpler and shorter.

APPENDIX I

SUMMARY OF SIMULATION SET-UP

Digital simulation of the manual rendezvous scheme is performed on an IBM 1130 computer, a desk-size machine featuring internal disk storage. This computer was chosen because 1) its dollar cost per hour is low enough to be feasible for completely dedicated use by a human operator whose reaction time is significant, and 2) its input-output lends itself to the kind of operation desired, i. e. insertion of an astronaut decision, computation, output of a computational result reflecting a change in the environment, evaluation of the result by the astronaut, and finally, a new astronaut decision, repeating the process. The structure of the FORTRAN program used in the simulation is designed to implement this conversational mode. The program is best described as consisting of a main inter-communication section which interfaces with more than 20 subroutines for the basic calculations.

The simulation is carried out by a trained operator, the "astronaut," working in a conversational mode with the computer. The computer keeps track of the exact position of target and spacecraft relative to inertial space, and to each other. It implements spacecraft maneuvers called for by the astronaut, suitably changing the orbital parameters of the spacecraft. It keeps track of light and dark periods, assuming the target to be visible only in the dark for purposes of sextant observations. It maintains the "star field," i. e. celestial coordinates, and gives the astronaut the sighting angles that would be observed between the line-of-sight to the target and the line-of-sight to selected stars.

The astronaut makes the human decisions. He decides the number of sightings to be made and the time at which each is taken. By interpreting the results of his sightings according to the manual guidance system, he decides what kind of flight plan to implement -- the direction, magnitude, and time of each burn -- and then "commands"

each burn by notifying the computer of his decision. Working close to real time, he leads the computer through the complete rendezvous mission. In short, the computer represents the environment, and the operator represents the astronaut's decision-making processes. In the simulation, as in the actual mission, the astronaut must achieve the rendezvous.

ACKNOWLEDGMENTS

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We wish to thank Mr. Archie P. Kelley, Jr. for his help in the study of the manual computation process.

NOMENCLATURE

A. Symbols

Cost	combined fuel-time cost criterion
k	cost-of-time weighting constant, knots/radian
r	shape parameter of relative in-plane trajectory
t	time
t_1	a desired measurement time
t_f	rendezvous time of mission as measured from $t = 0$ hrs
T	period of reference orbit
x	downrange component of relative displacement, nm
y	out-of-plane component of relative displacement, nm
z	upward component of relative displacement, nm
δ	in-plane angle (angle between x-axis of LTCS and projection of line-of-sight onto plane of reference orbit); $\cot \delta \equiv x/z$

A. Symbols (continued)

Δt	time interval between first and last measurements of star-sight averaging process
ΔV	velocity change magnitude, knots
ϵ	eccentricity
η	dimensionless vertical velocity; $\dot{z}/\omega z$
ξ	dimensionless horizontal velocity; $\dot{x}/\omega x$
σ	out-of-plane angle (angle between line-of-sight and its projection onto the reference orbit plane); $\tan \sigma \equiv y/(x^2 + z^2)^{\frac{1}{2}}$
τ	dimensionless time interval (radians) between a point on a trajectory and the nearest point at which apogee is reached
ω	angular velocity of reference orbit, radian/hour

B. Abbreviations

A_p	apogee passage time
3B	three-burn maneuver calculation
DB	dark period begins
DE	dark period ends
hrs	hours
IPNS	in-plane normalized state
LTCS	local tangent-to-the-flight path coordinate system
mr	milliradians
nm	nautical miles
PC	plane crossing time
P_e	perigee passage time

B. Abbreviations(continued)

rad	radians
SP	state projection
3	three-instant out-of-plane state determination method (one of three possible techniques for determining state using the manual system)

C. Other Notations

(')	differentiation with respect to t
0	subscript, initial conditions
●	measurement
⊗	out-of-plane thrust maneuver
●→	in-plane thrust maneuver

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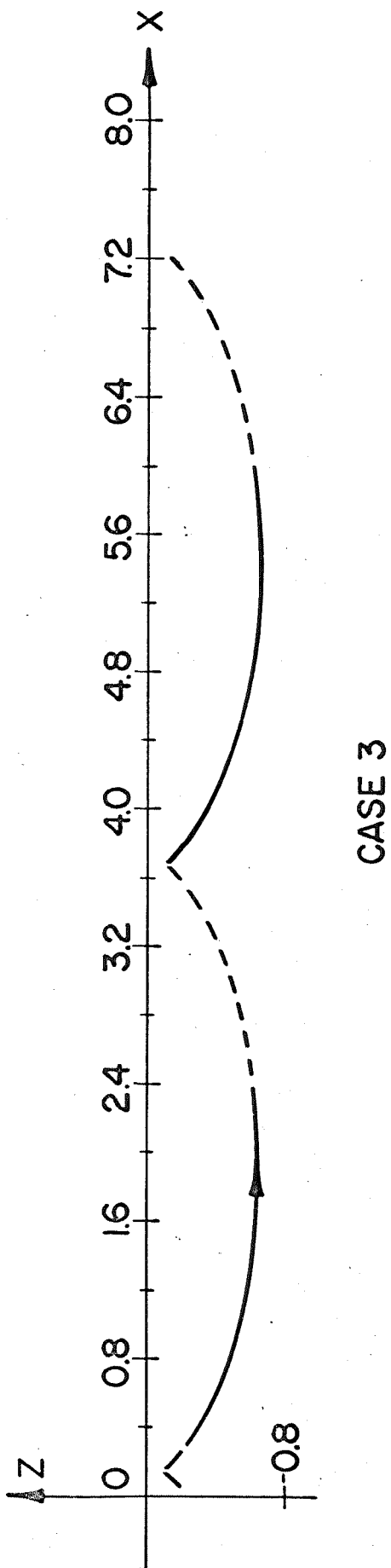
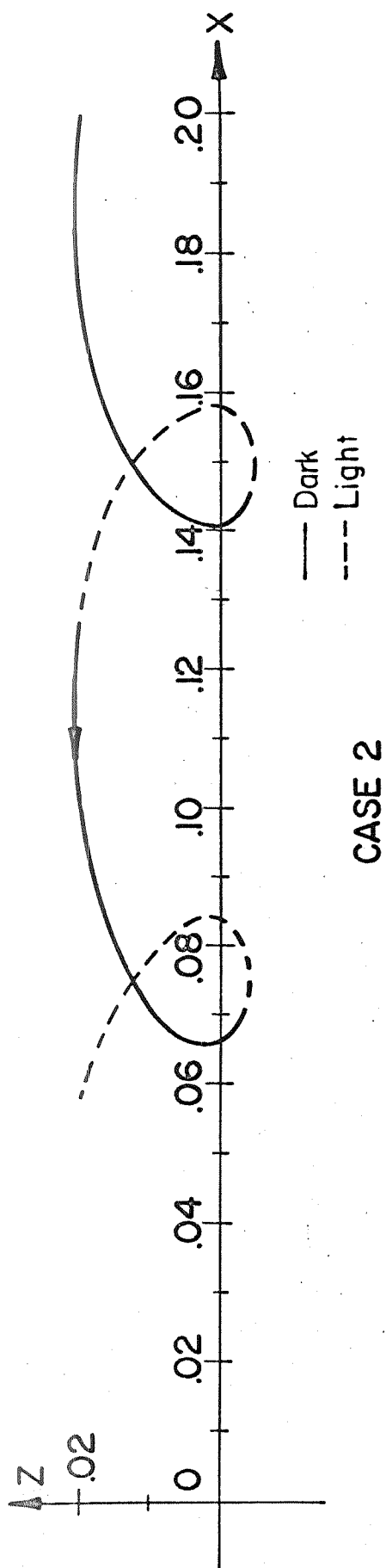
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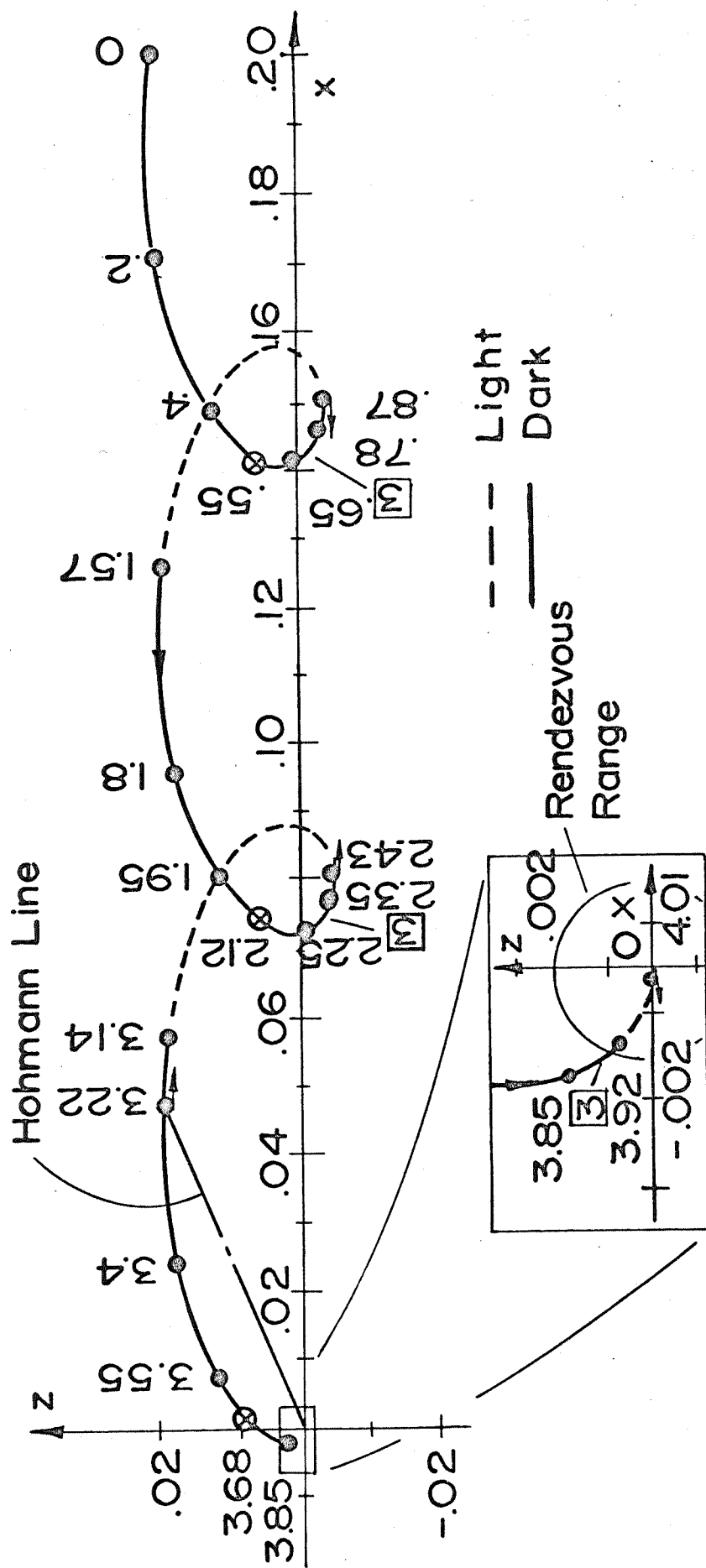
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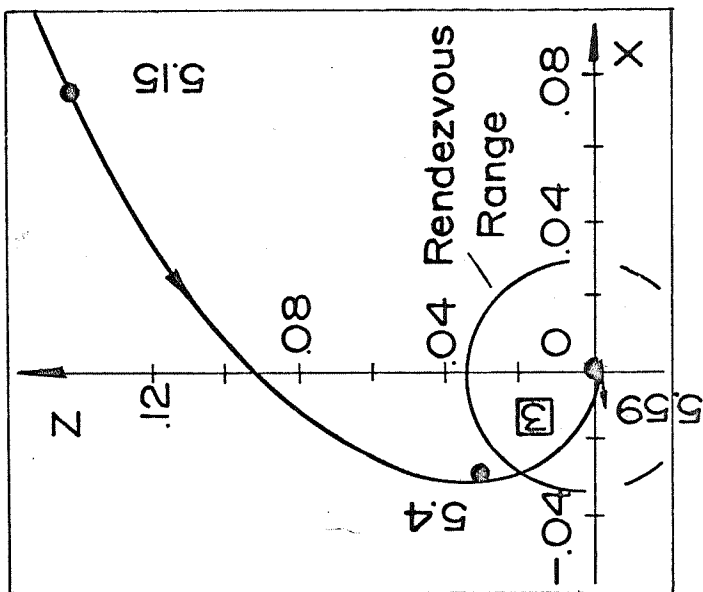
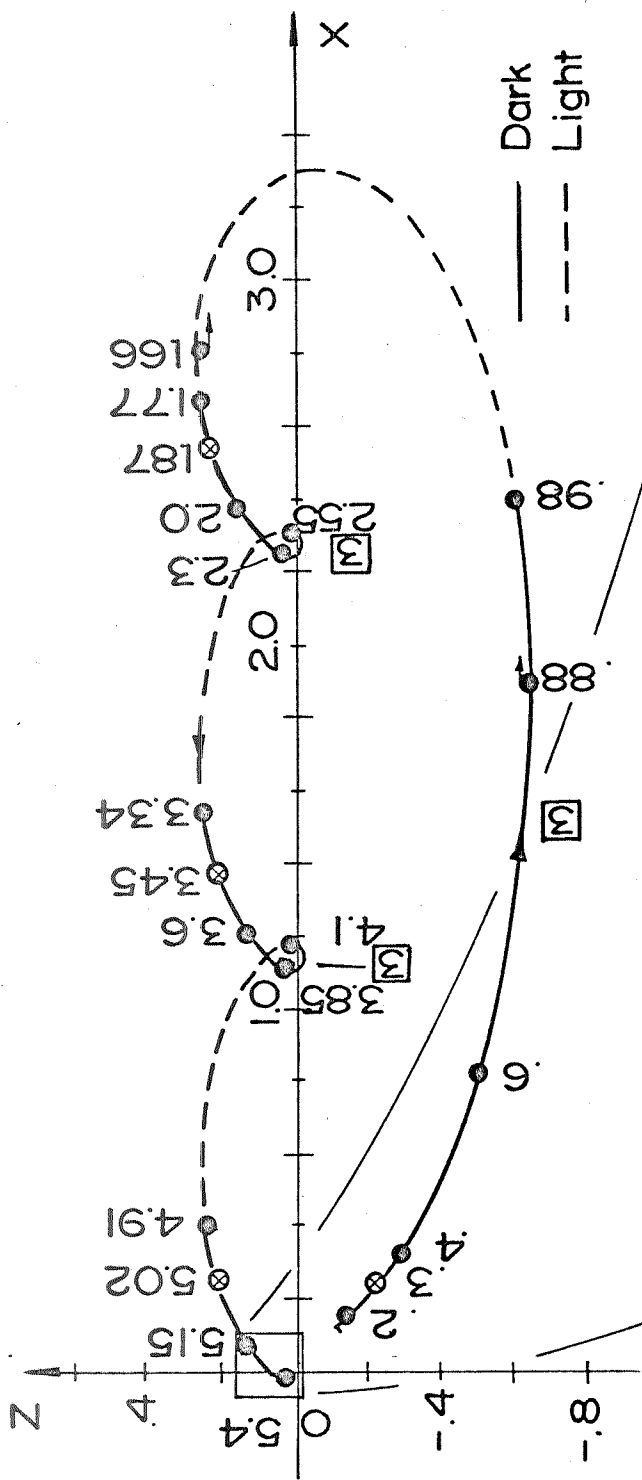
FIGURE CAPTIONS

<u>Figure</u>	<u>Caption</u>
1	Test Case Initial Conditions
2	Reference Mission - Case 2
3	Reference Mission - Case 3
4	Mission Performance Relative to Separation Range
5	Mission Performance Relative To Reference Orbit Eccentricity
6	Mission Performance Relative To Error In Burn Direction
7	Mission Performance Relative To Bias in Burn Magnitude
8	Mission Performance Relative to Measurement Error
9	Mission Performance Relative to Star Sight Averaging Error
10	Mission Performance Relative to Error In Knowledge of Interceptor Orbit Period
11	Comparison of Error and Reference Mission - Case 2 - Eccentricity = .02
12	Comparison of Error and Reference Mission - Case 2 - Burn Direction Error = 1.15°
13	Comparison of Error and Reference Mission - Case 3 - Eccentricity = .03
14	Error Mission - Case 2 - Multiple Error Sources
15	Error Mission - Case 3 - Multiple Error Sources
16	Astronaut Time-Activity Chart for Case 3

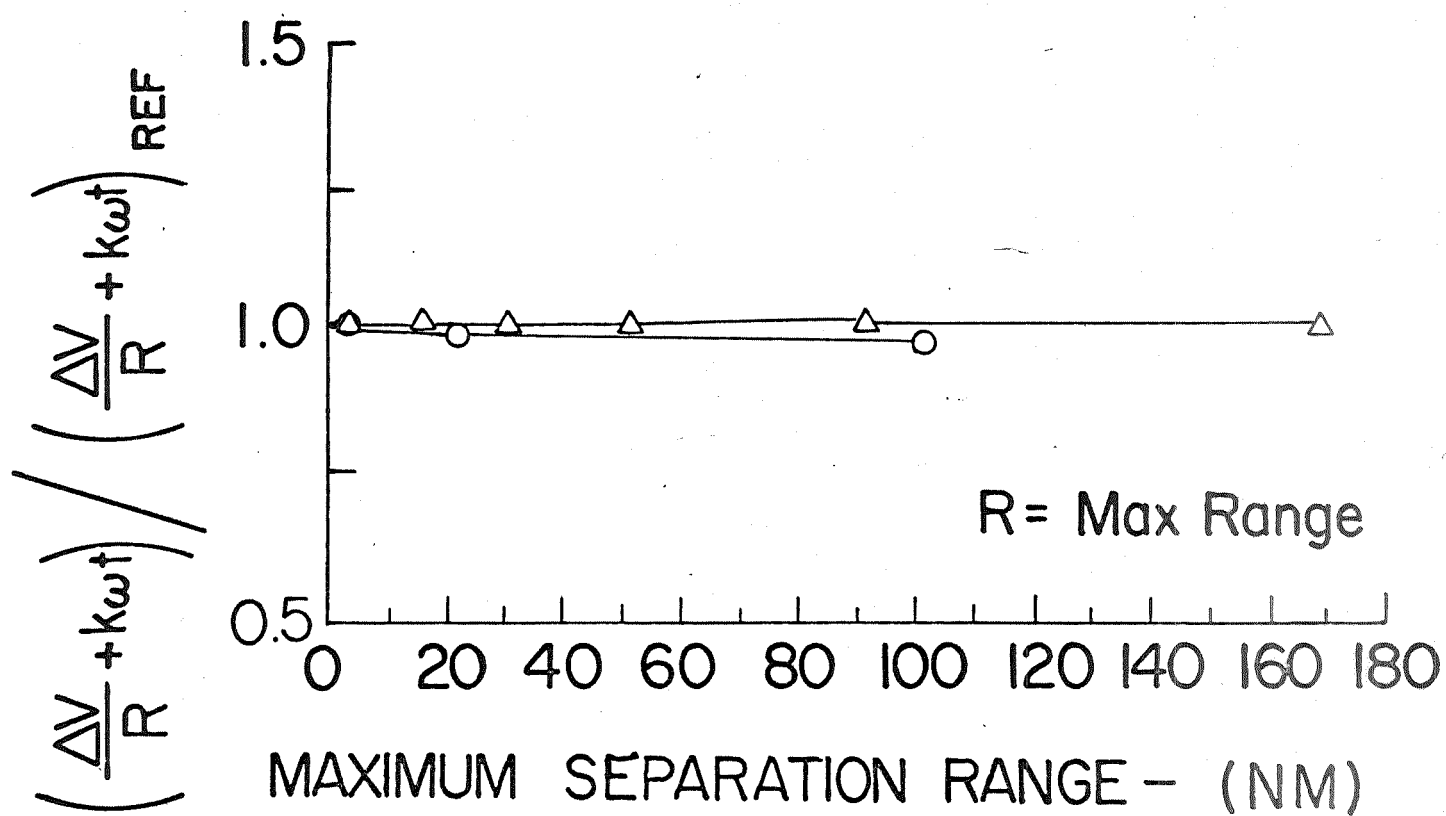
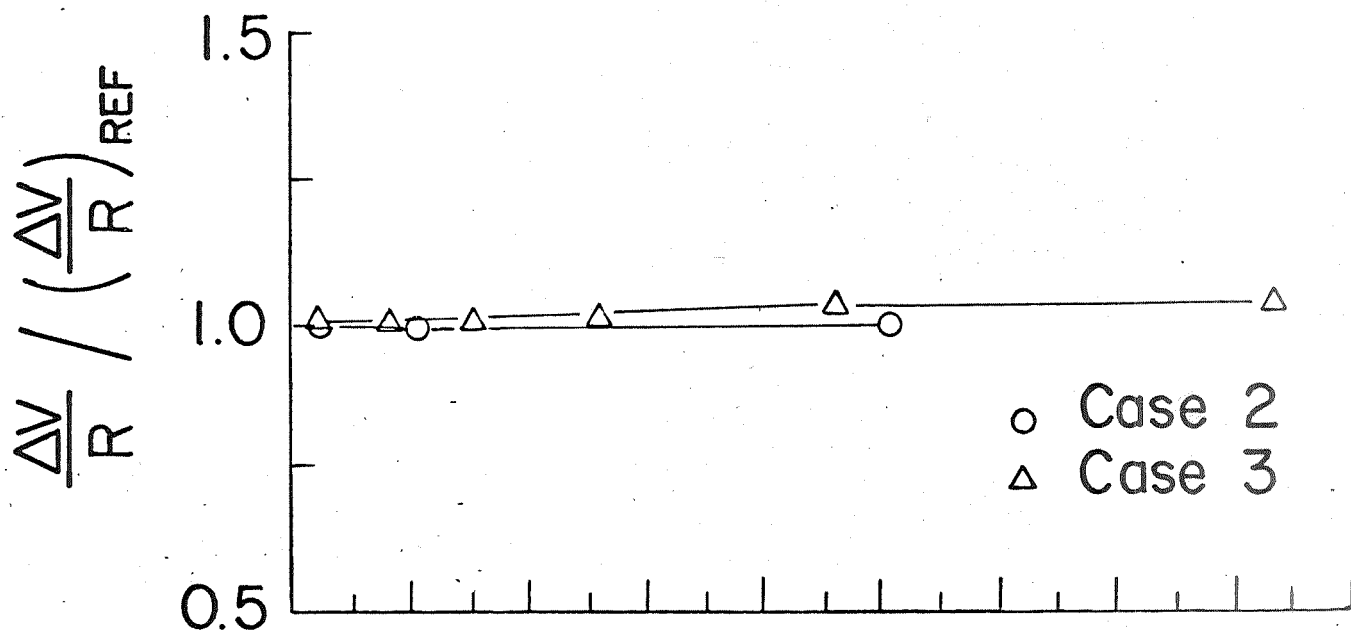


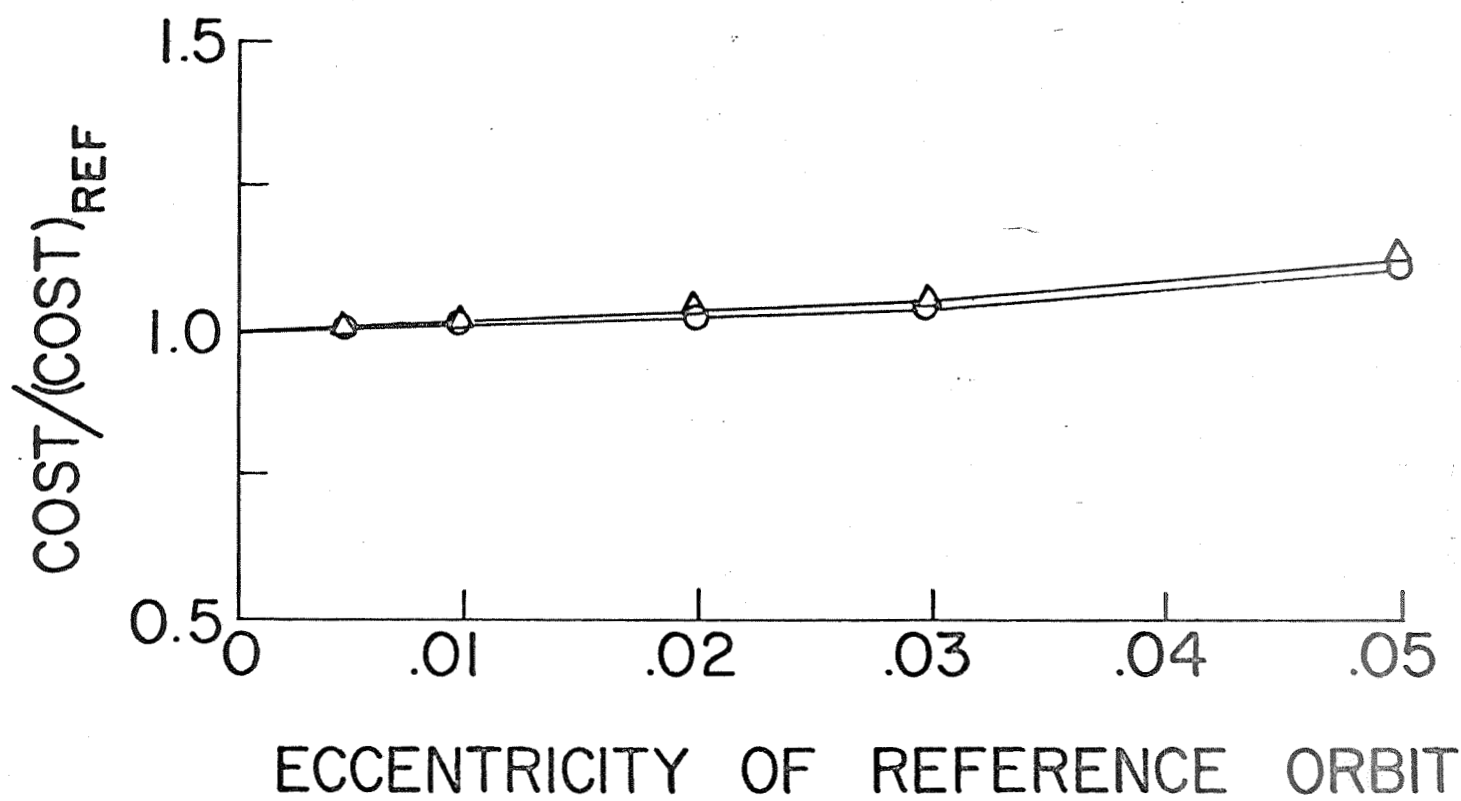
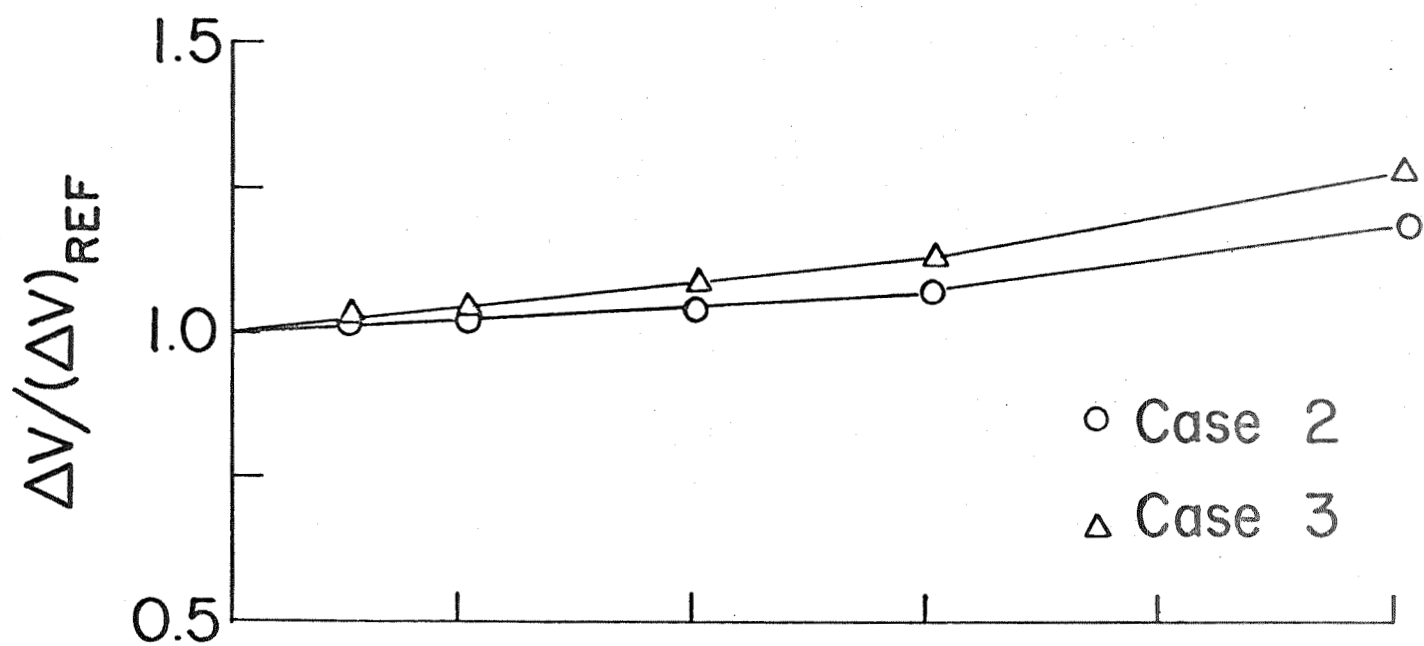


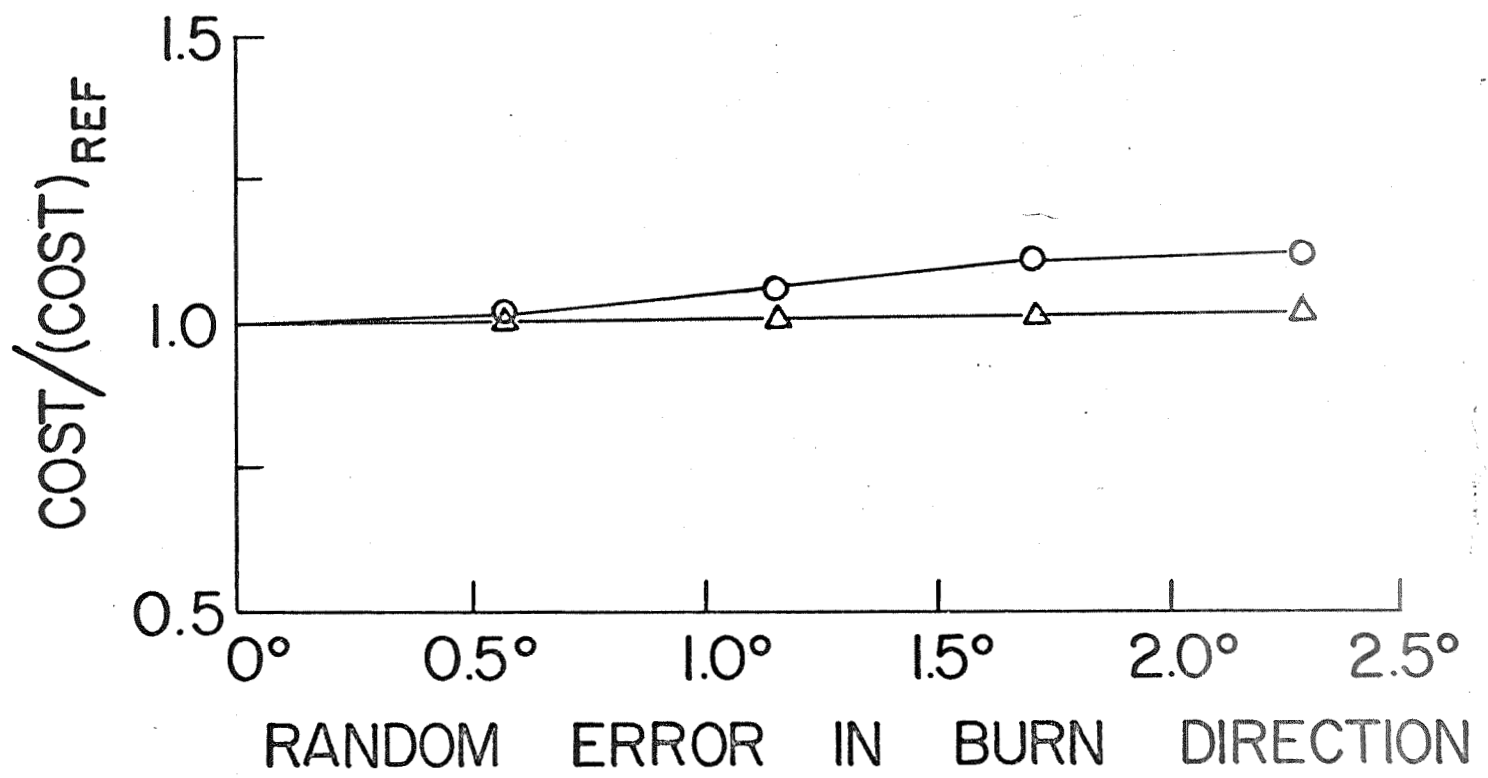
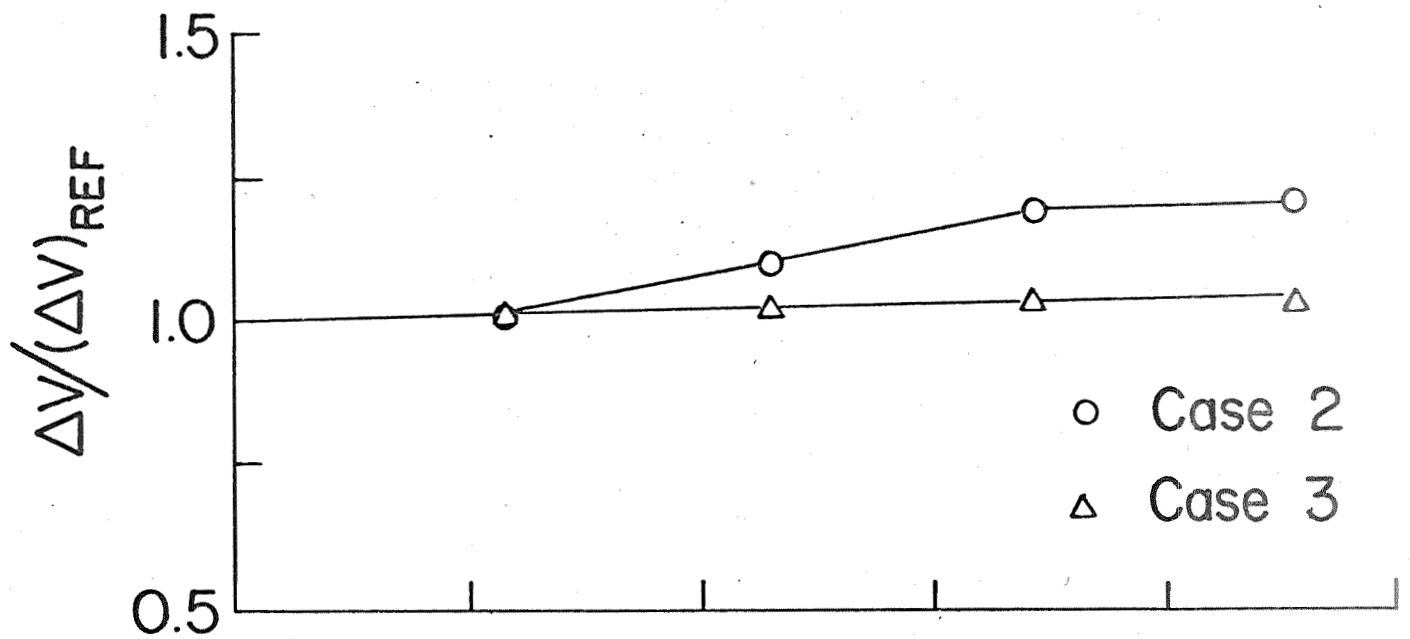
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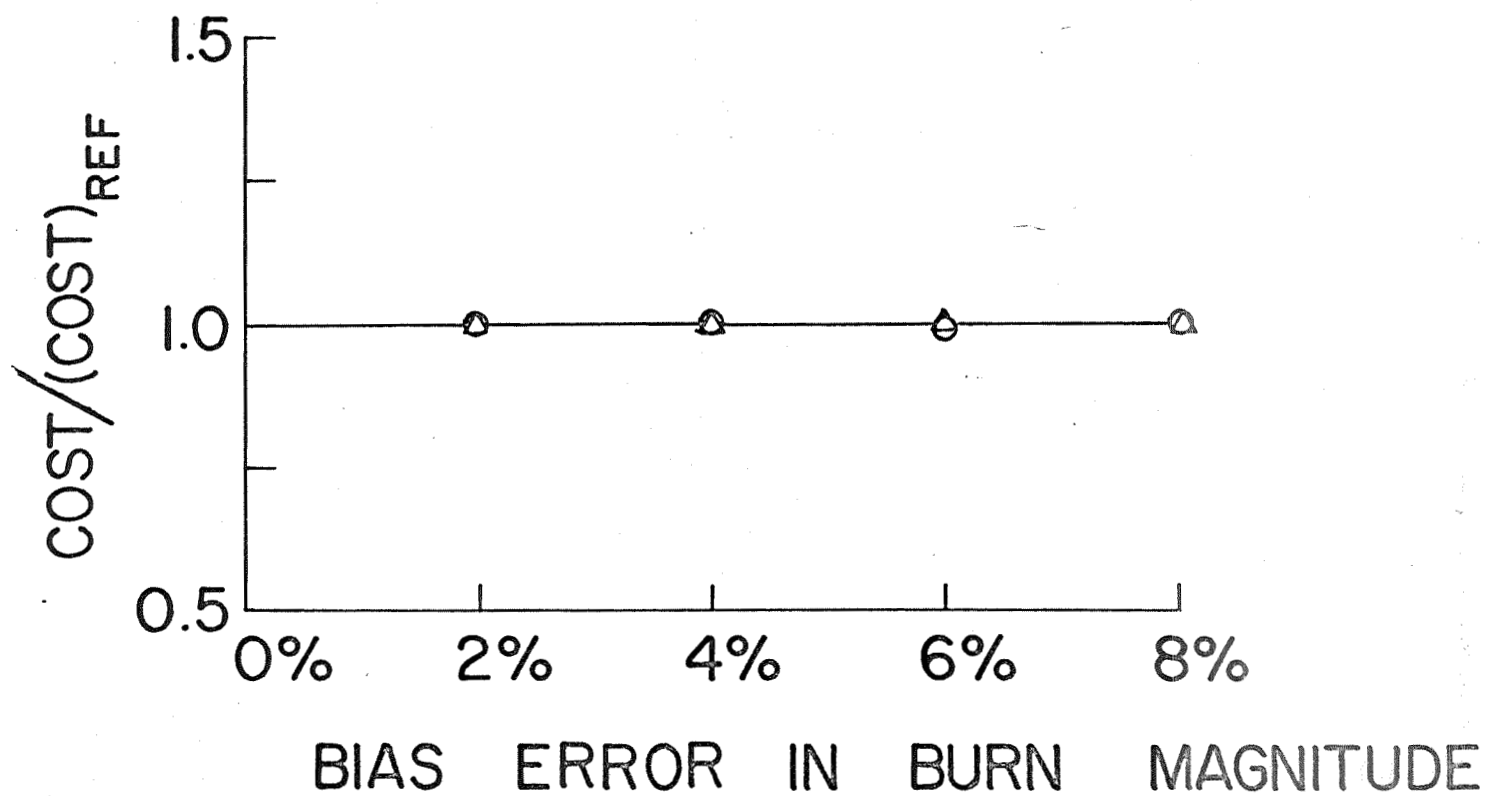
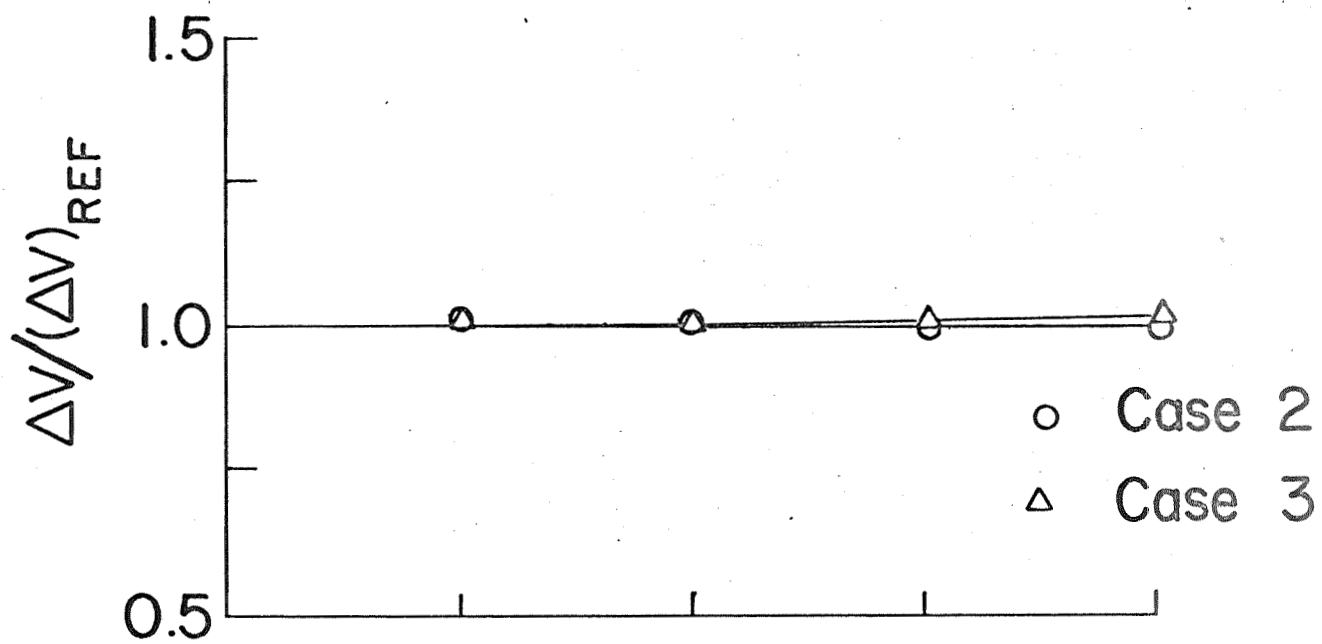


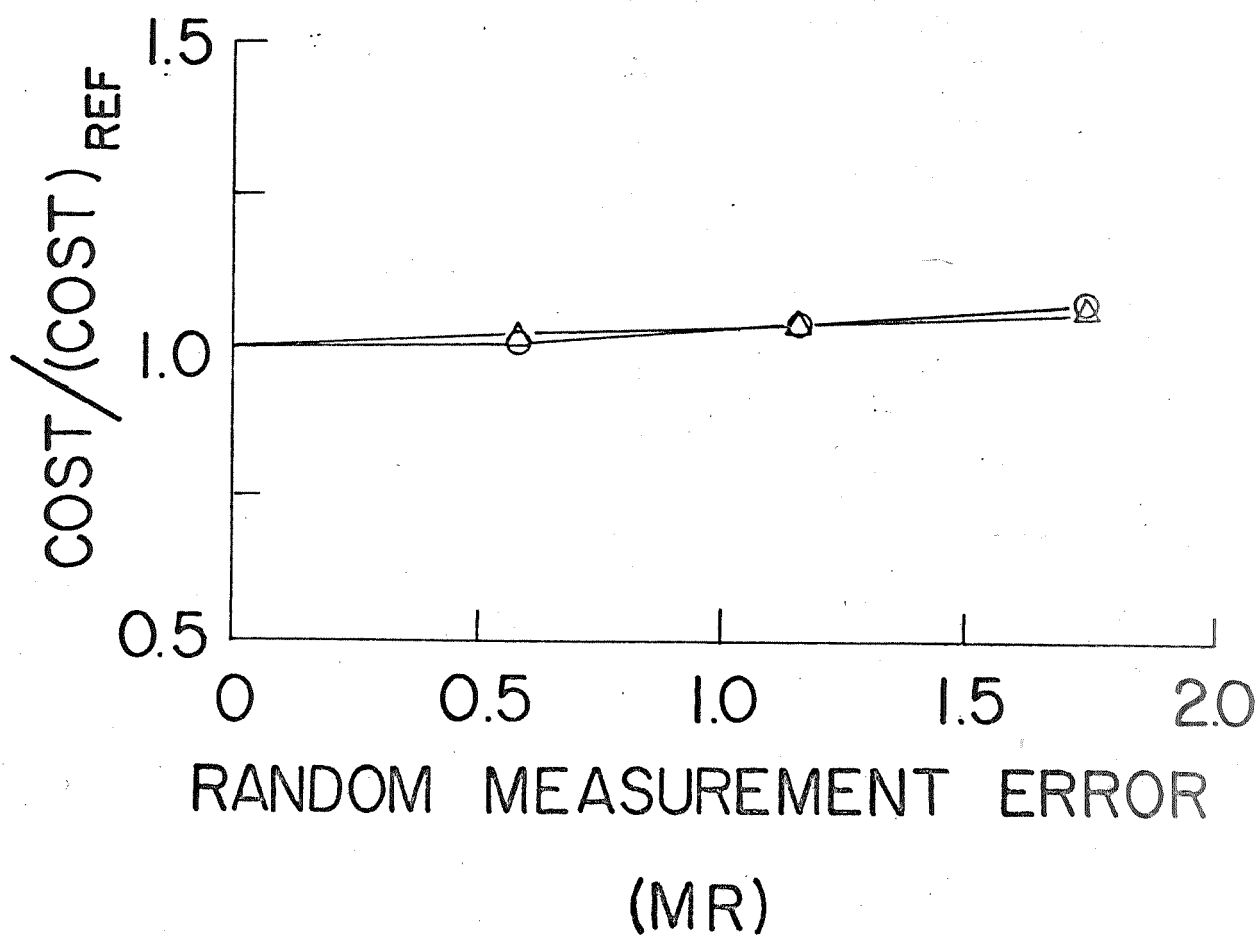
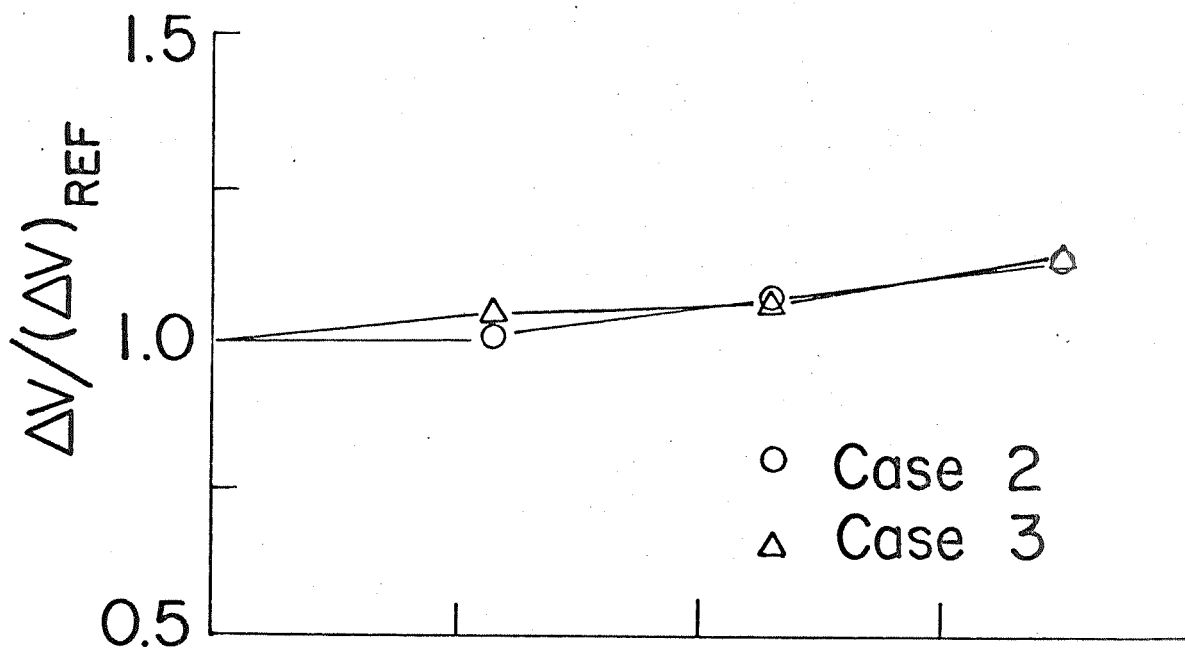
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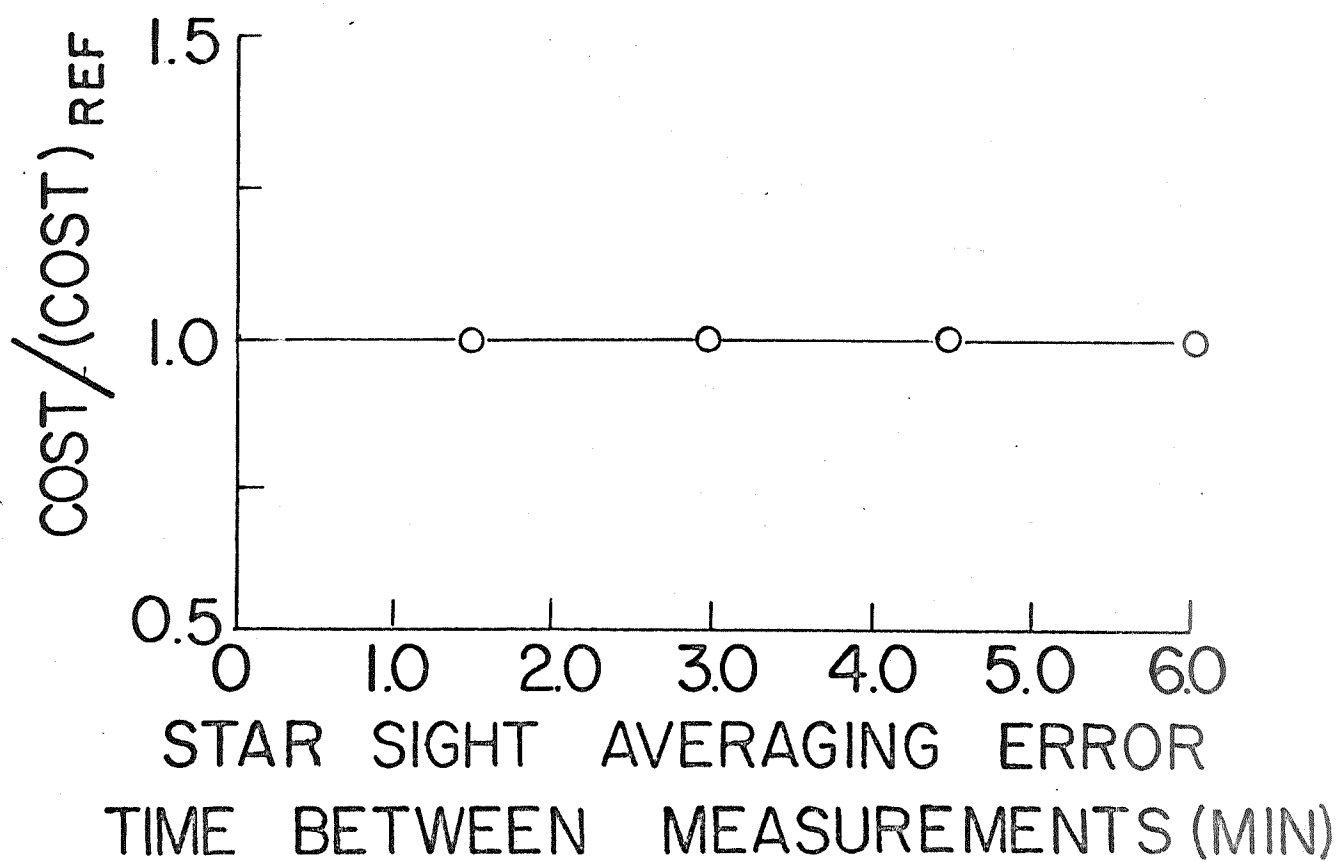
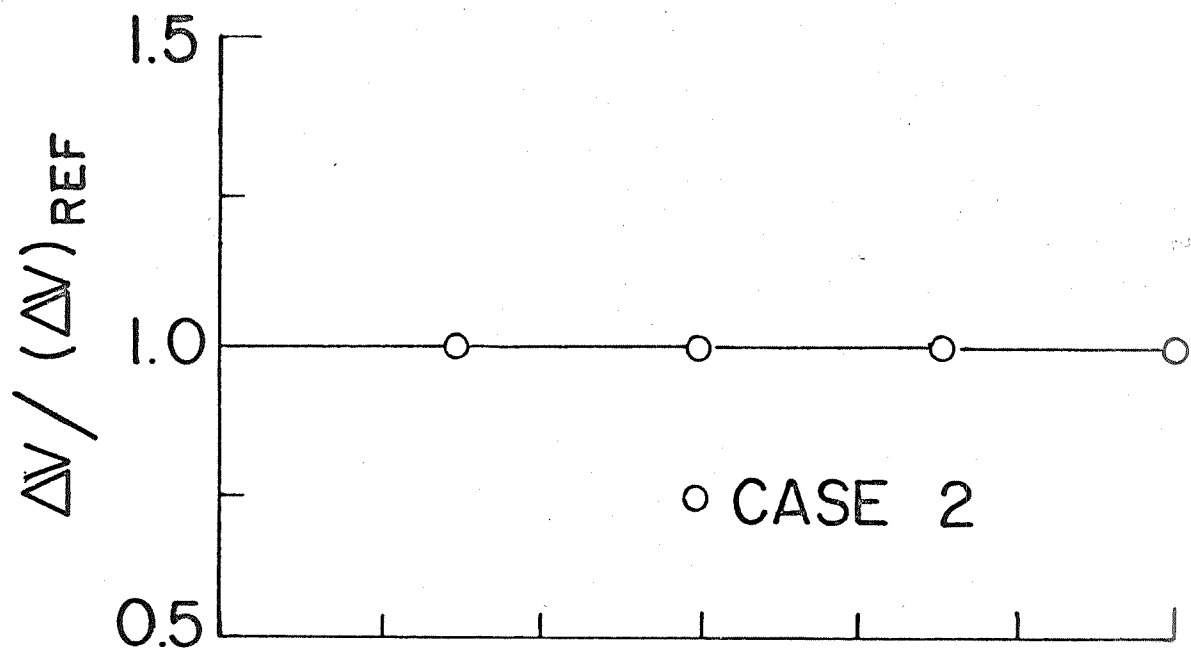


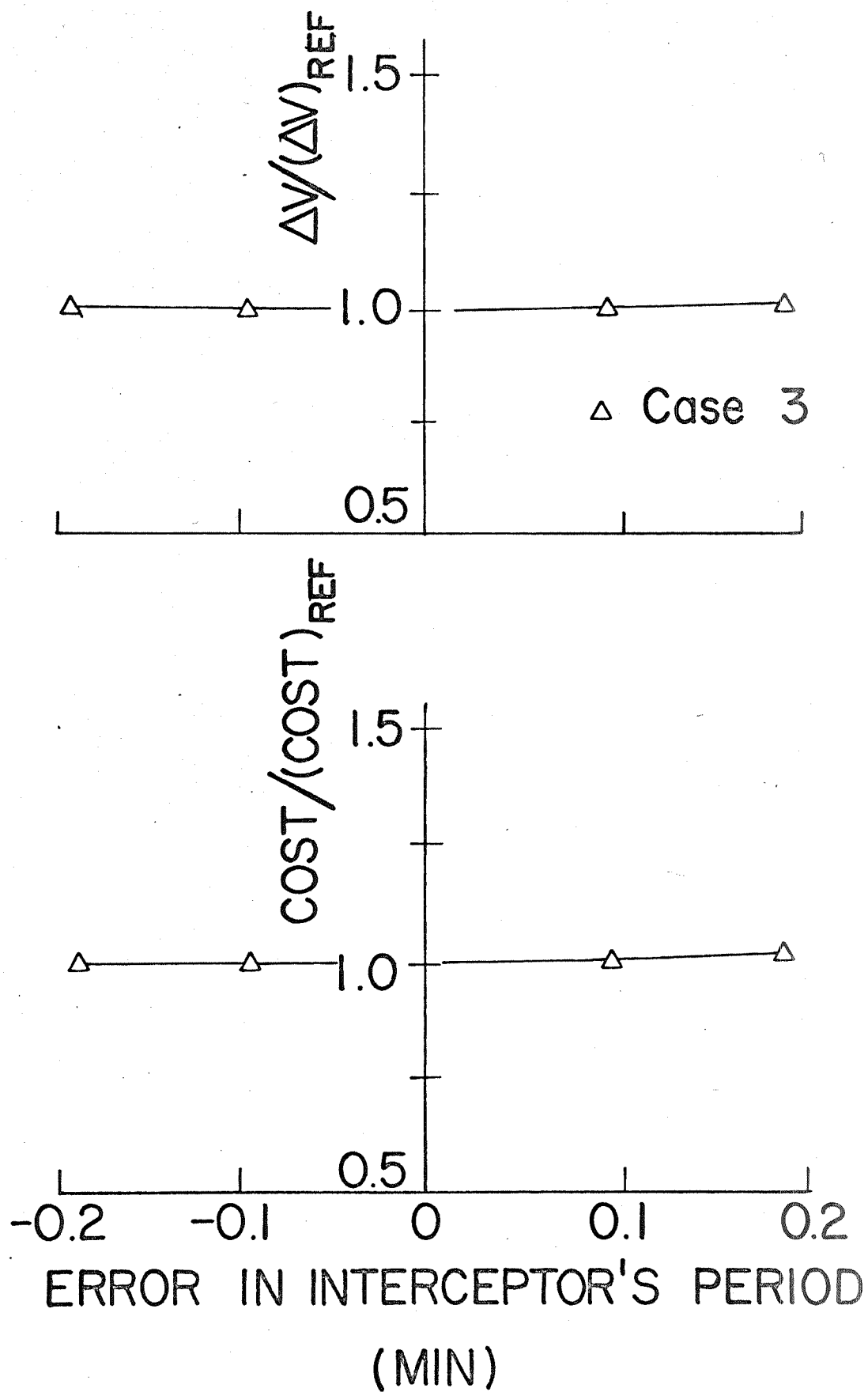


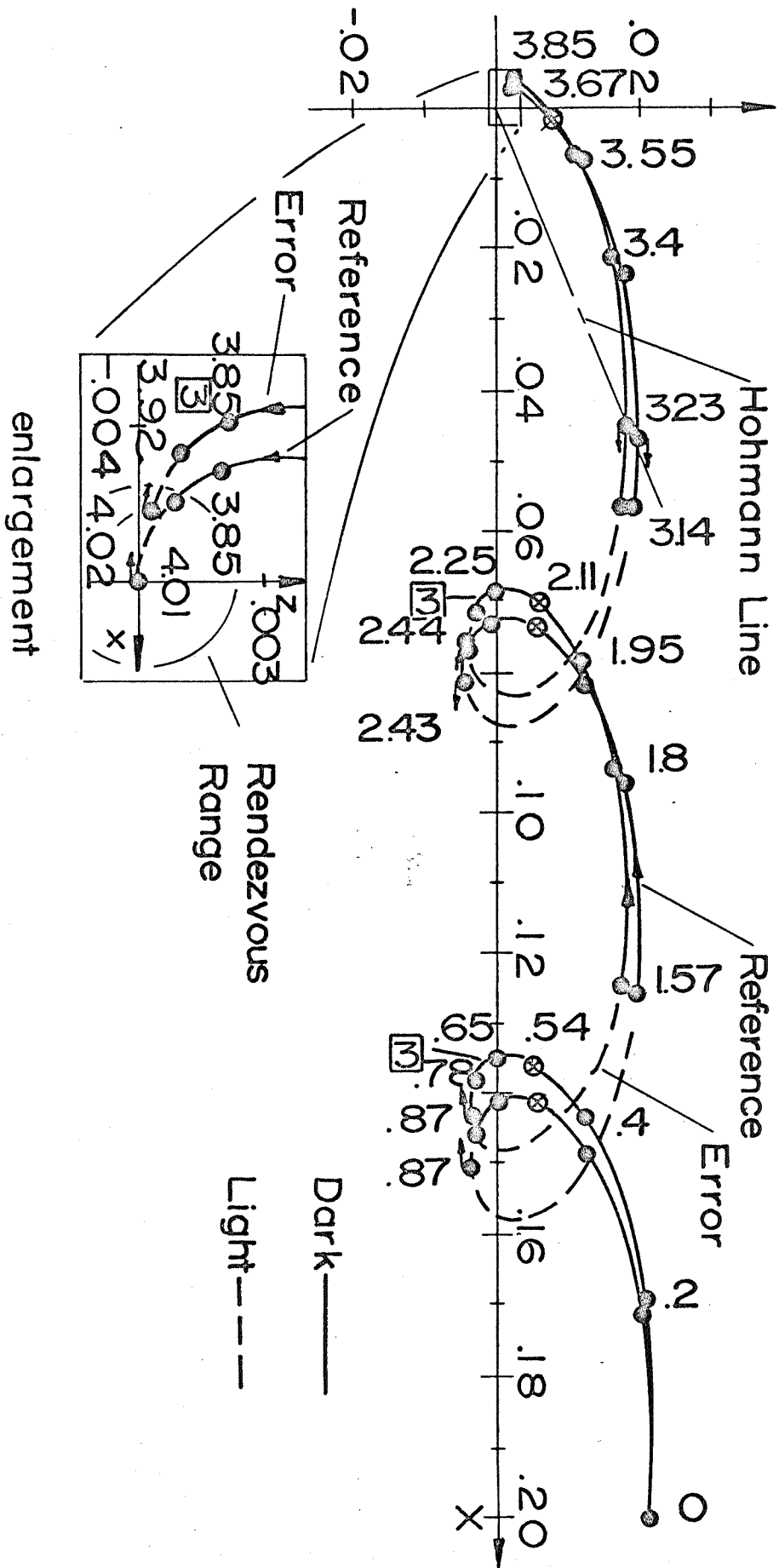


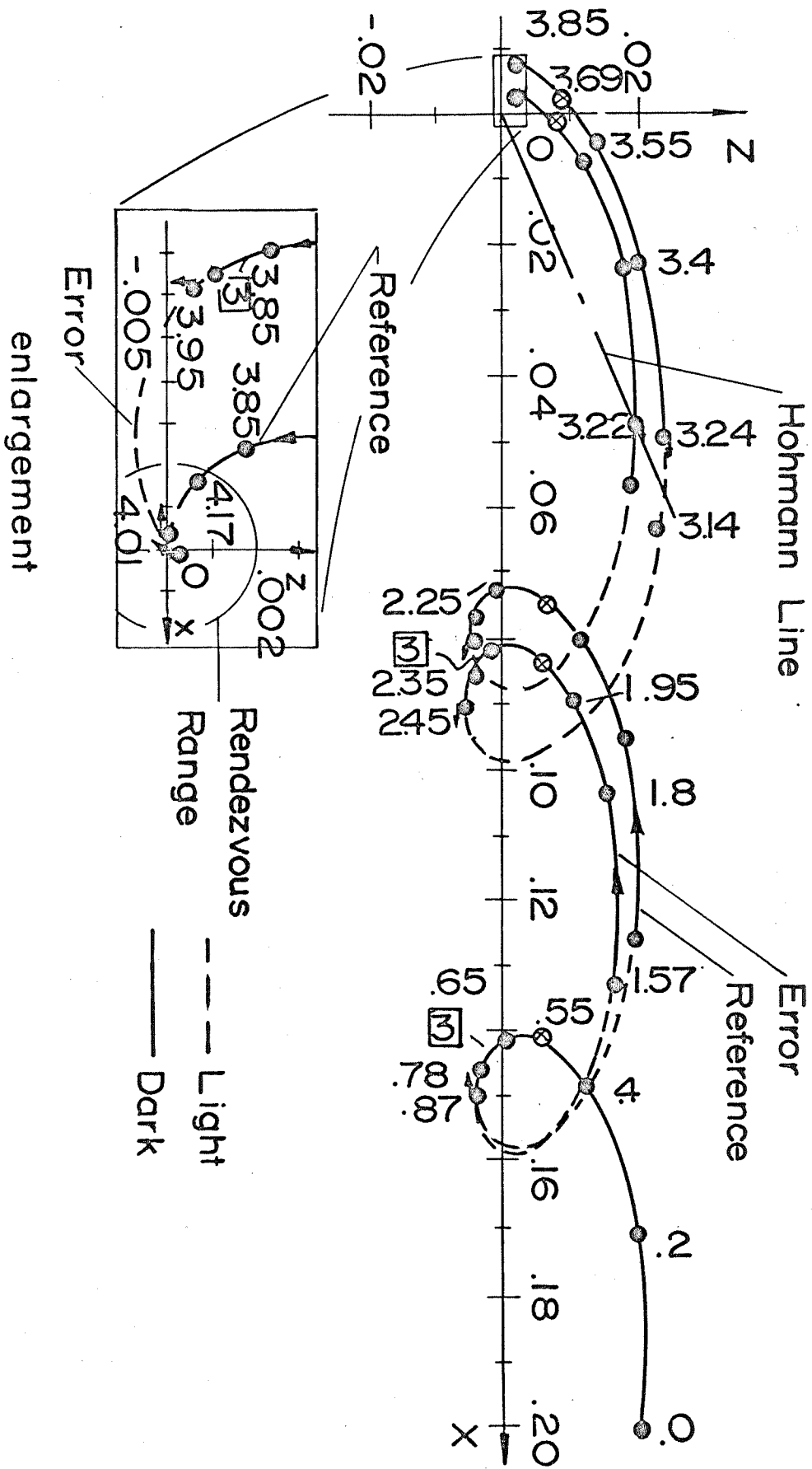


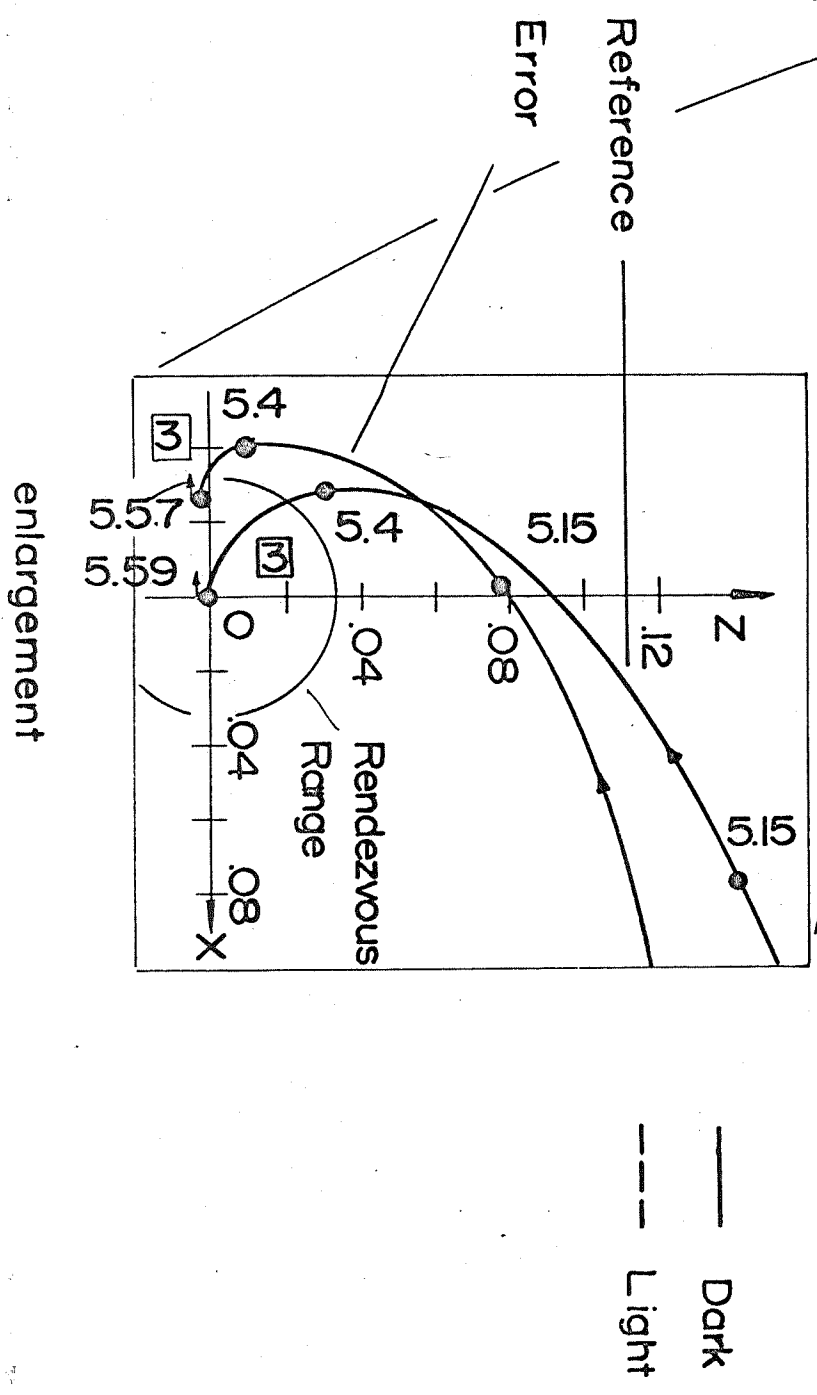
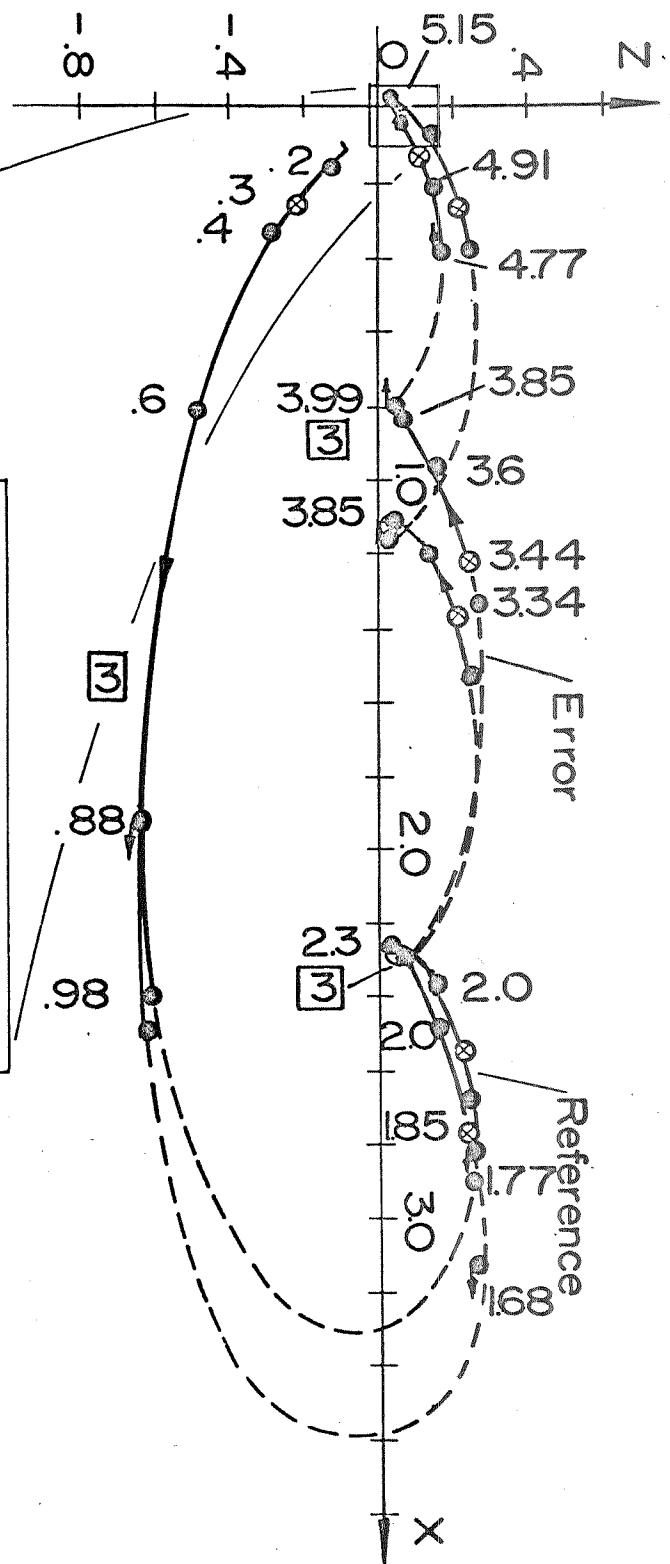


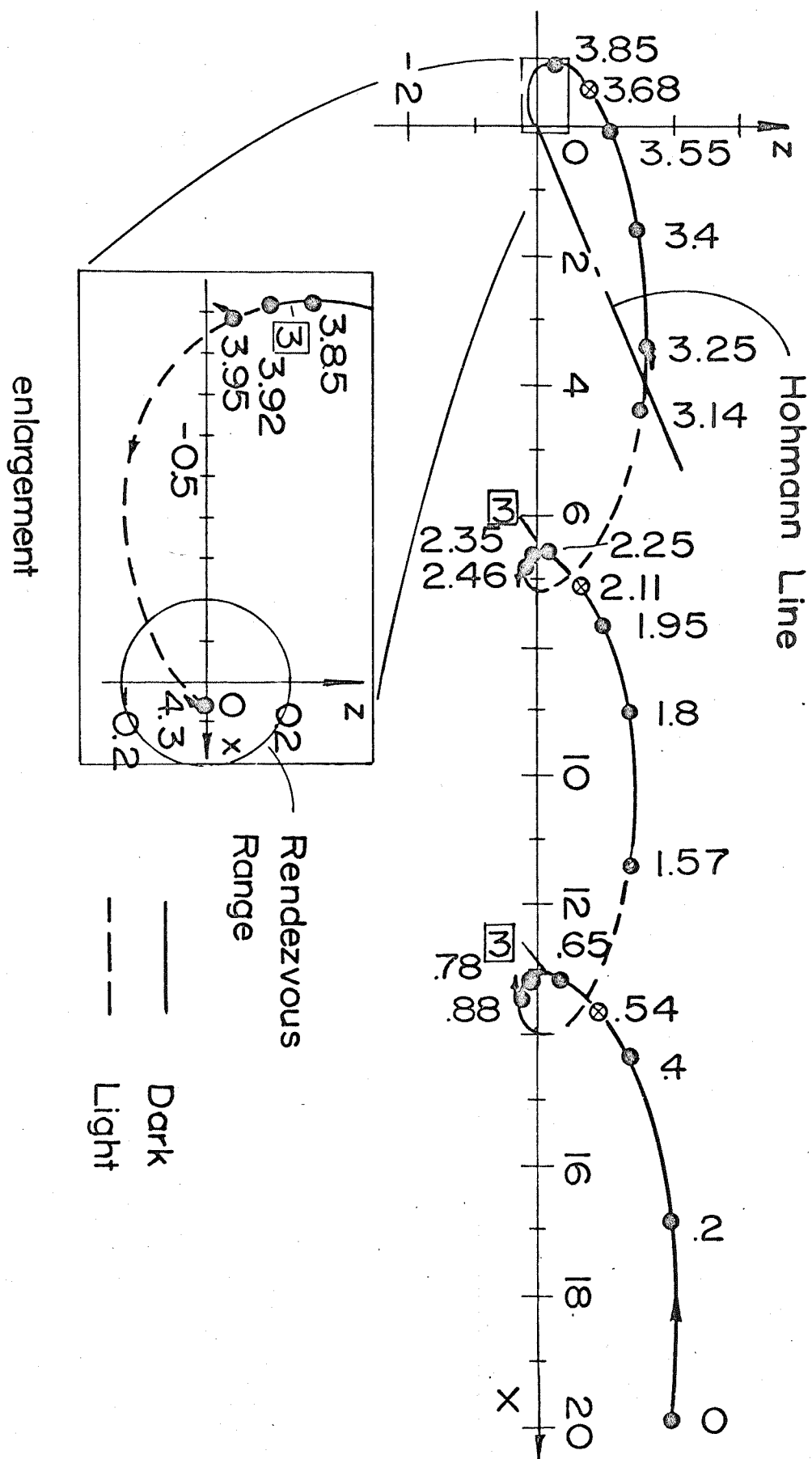


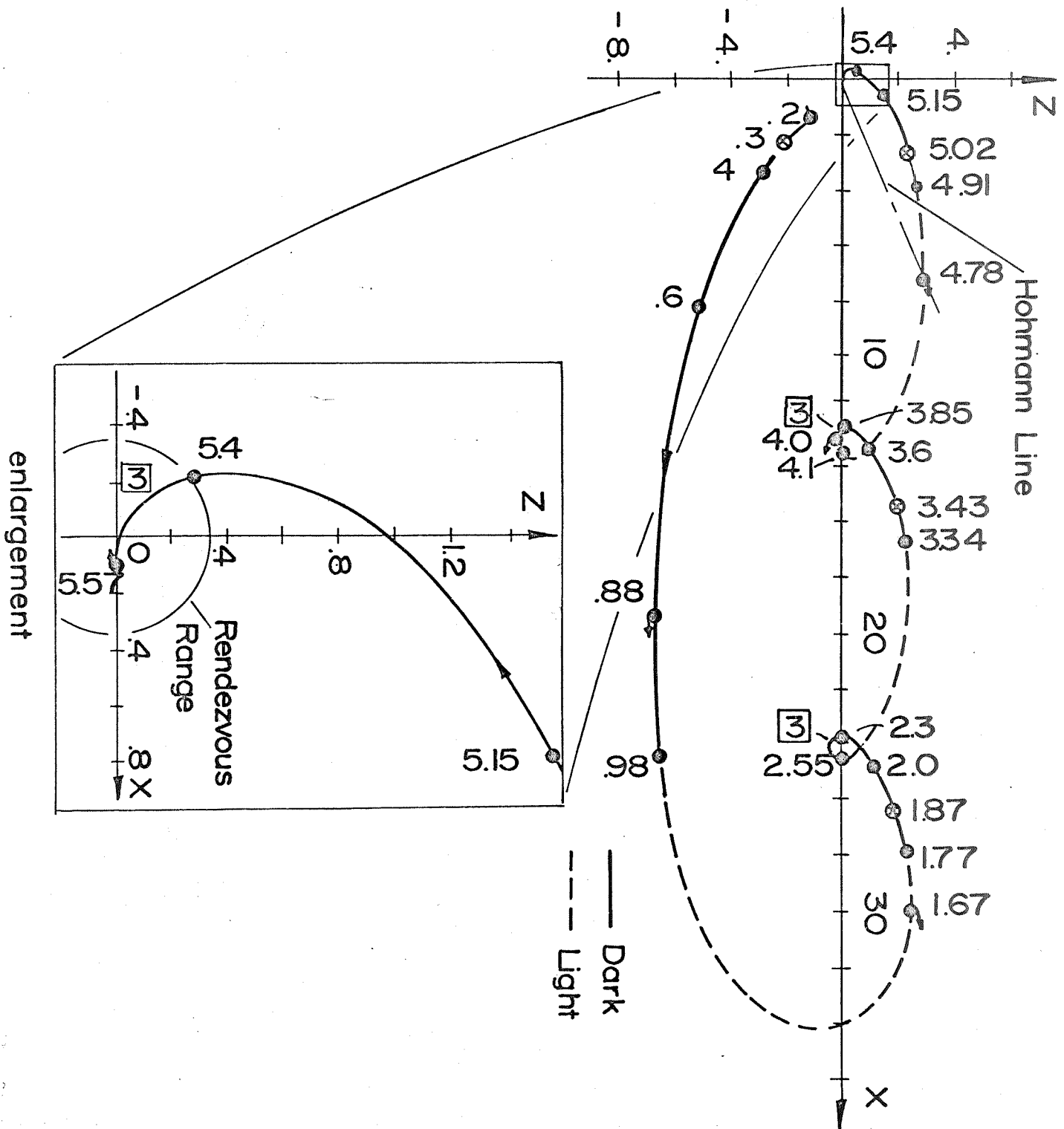




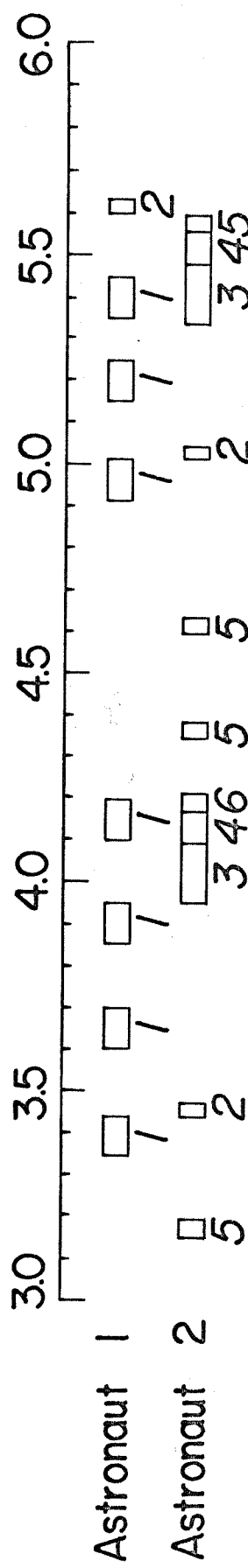
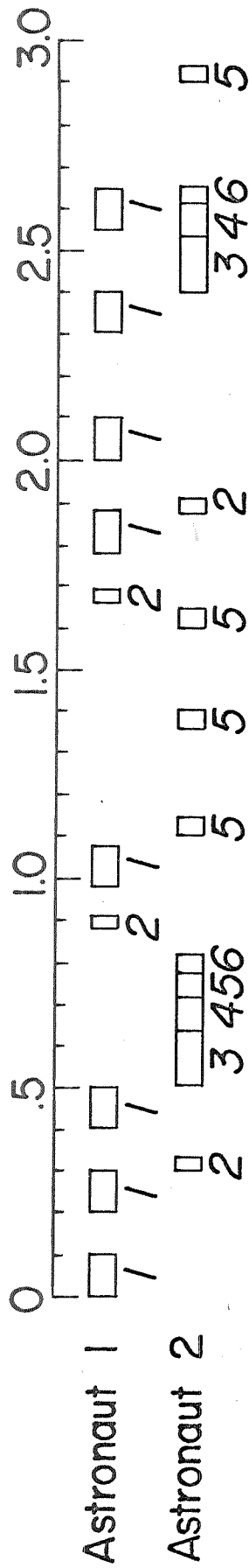








TIME (hrs)



Legend :

- 1 Take Measurement & Calculate δ, σ
- 2 Perform Thrust Maneuver
- 3 Compute In-plane Normalized State, Range
- 4 Compute State
- 5 Project State
- 6 Compute Burn Vector Components

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A Simple Linear Approximation for Perturbed Motion about Moderately Elliptic Orbits

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IN this Note, the linearized perturbation equations for a circular reference trajectory are applied to elliptic reference trajectories of small eccentricity. The concept is appealing because the closed form solution for the circular reference is so much simpler than the solution for the elliptic case. A numerical example indicates that the accuracy of the circular reference solution is more than adequate for many practical applications. An analysis is outlined to extend the results of the numerical example.

The linear, constant-coefficient differential equations that describe the perturbed motion of a body P relative to a reference body O in a circular orbit are well known.¹⁻³ The solution to these equations, which for brevity will be called the circular solution, is usually written in terms of coordinates in a local vertical coordinate system centered at O. The circular solution, which contains sines and cosines and terms linear in time, is simple enough to be calculated by hand. It has di-

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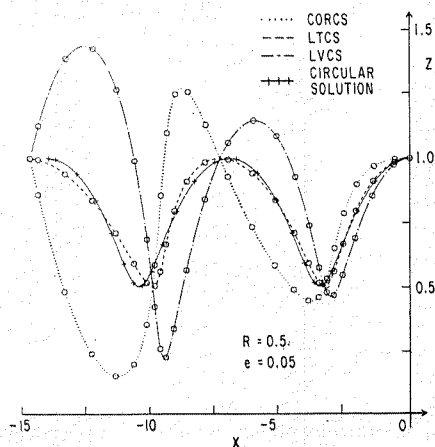


Fig. 1 Comparison of in-plane trajectory of perturbed body in three candidate coordinate systems with circular solution.

rect geometrical interpretation in terms of a point on an interior or exterior radius of a rolling disc. This simplicity makes the equations attractive for use in many applications. In Ref. 3, for example, they form the basis for constructing a manual guidance scheme for rendezvous, in which the non-maneuvering target is the reference body and the interceptor is the perturbed body. This Note extends their use by showing that an interpretation of the coordinates exists for elliptic reference orbits such that the circular solution adequately defines the observed perturbation behavior.†

Three different coordinate systems were examined for expressing the small perturbations from the elliptic reference orbit. These were designated the local vertical (LVCS), the constant orbital rate (CORCS), and the local tangent (LTCS) coordinate systems. All three are right-handed Cartesian systems centered at O, where the y axis is out of plane, the z axis is more or less in the outward radial direction, and the x axis is more or less in the direction of motion. More precisely, in LVCS, the x axis is normal to the radius vector from the force center to O. In CORCS, x is initially parallel to the x axis of LVCS, but then rotates at a constant rate relative to inertial space, equal to the mean motion of the reference orbit. In LTCS, the x axis is tangent to the reference orbit at O.

The accuracy of the circular solution in describing the motion about an elliptic reference orbit was tested on the digital computer. The simulation used exact equations of motion to propagate the absolute position of both the reference and perturbed body. Test cases were selected spanning the space of all trajectories inherent in the circular solution, in the sense that they cover the full range of the in-plane shape parameter r , defined by $r = z_a/z_b$ where z_a and z_b are the extreme values of z , z_a being the one of smaller absolute value. The change in semimajor axis δa of the perturbed trajectory is linearly related to r by the formula $0.5z_b(r + 1)$. Since the y motion is the same in all three coordinate systems and since it is decoupled from the in-plane motion, only the in-plane or x - z solution is germane.

Samples of the results are shown in Figs. 1-3. Figure 1 shows the exact relative in-plane motion of the perturbed body as seen in each of the three coordinate systems. In addition, the circular solution appears for comparison. A value of 0.05 was used as the eccentricity of the reference orbit. One unit is 0.001 nautical mile (nm), and Earth orbits having a 3700

nm semimajor axis are assumed. Tick marks are placed on the trajectories $\frac{1}{2}$ of a reference orbit period apart.

First, it is seen that the circular solution very nearly matches the exact solution in the LTCS. Second, it is noted that the match is better for LTCS than for either LVCS or CORCS. Although not apparent from the data presented here, it was found that the deviation, for given conditions, between the circular solution and the exact trajectory is linear with eccentricity over the range of values tested.

The eccentricity error of the circular orbit equations in a given coordinate system is defined as the value of relative position (x, z) at a given time computed from these equations, minus the exact value of relative position. The eccentricity error Δx and Δz for the three coordinate systems is plotted vs. time t in Figs. 2 and 3, respectively. In order to reduce second order effects, a value of eccentricity equal to 0.01 was chosen for these plots; all other conditions are the same as were used for Fig. 1.

Examination of the complete set of these curves⁷ reveals that for any value of r , the x -error at the end of an integral number of orbit periods is nearly the same in all three coordinate systems, but varies linearly from zero to a maximum as r varies from -1 to 1 . Hence, the x -error is proportional to δa . In addition, while the z -error of LVCS and CORCS is finite, one being roughly the negative of the other, the z -error of LTCS is essentially zero. The practical elimination of the z -error is, in fact, the key to the improvement of LTCS over the other two coordinate systems.

Straight lines on the x - and z -error plots approximately bound the errors. These lines are drawn from the origin through the points $[t = T, \Delta x \text{ or } \Delta z = \pm e \cdot x(T)]$ where T is the period and e is the eccentricity of the reference orbit. Thus, the error is bounded by a function that grows linearly with time and is proportional to the reference orbit eccentricity. These bounds do not pertain to Δz in LTCS which is essentially zero, as noted previously.

It was pointed out by a reviewer that the proportionality of the error to e and δa , the equality of the x - and z -error bounds and their linearity with time, the apparent cosine and sine behavior in Figs. 2 and 3 respectively, and the essentially zero magnitude of the z -error in LTCS can be summarized in the equations

$$\text{LVCS} \quad \Delta x = -kt(\delta a)e \cos f \quad (1)$$

$$\Delta z = -kt(\delta a)e \sin f \quad (2)$$

$$\text{LTCS} \quad \Delta x = -kt(\delta a)e \cos f \quad (3)$$

$$\Delta z = 0 \quad (4)$$

where f is the true anomaly. Furthermore, he outlined the derivation of these equations and evaluated the constant k from the following theoretical considerations. The primary error in adapting the equations of circular motion to an orbit that is slightly elliptical is caused by the secular terms in the equations. It is appropriate to examine the full state transition

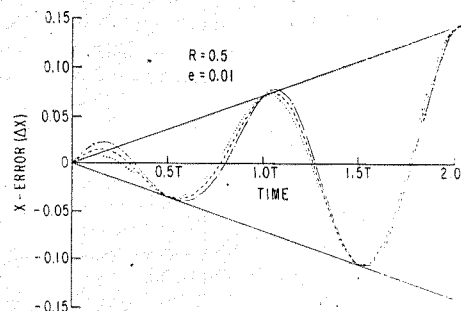


Fig. 2 The eccentricity error of the circular reference orbit solution in the three candidate coordinate systems: x axis.

† It should be pointed out that the linearized perturbation equations of motion in an elliptic reference orbit have been solved in closed form.⁴⁻⁶ However, the elliptic solution is more complicated than the circular solution, and hence the applicability of the latter is investigated.

§ Complete results appear in Ref. 7.

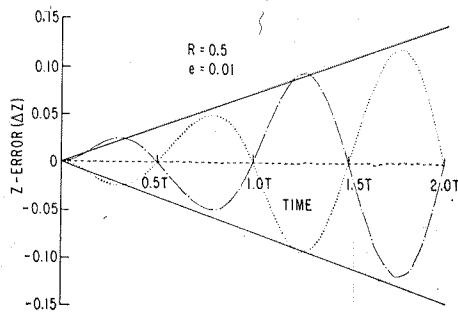


Fig. 3 Eccentricity error of the circular reference orbit solution in the three candidate coordinate systems: z axis.

matrix for an elliptic reference orbit to determine the effect of a small value of eccentricity on these terms. The secular terms are found to be linear in time and also linear in $\delta a/a$. In the local vertical coordinate system (LVCS) of this Note, the secular terms in position variation are

$$x = -1.5V_x t(\delta a)/a \quad (5)$$

$$z = -1.5V_z t(\delta a)/a \quad (6)$$

where

$$V_x = \mu(1 + e \cos f)/h \quad (7)$$

$$V_z = \mu e(\sin f)/h \quad (8)$$

In the local tangent coordinate system (LTCS) the corresponding terms are

$$x = -1.5V_x t(\delta a)/a \quad (9)$$

$$z = 0 \quad (10)$$

where

$$V = \mu(1 + 2e \cos f + e^2)^{1/2}/h \quad (11)$$

V_x and V_z are components of reference velocity along the x and z axes, V is the reference velocity magnitude, μ is the gravitation parameter, and h is the angular momentum. To first order in e , the secular errors due to neglecting e are then given by Eqs. (1-4) with $k = 1.5\mu/ha$.

In summary, the circular orbit equations can be used to describe perturbed motion about a moderately elliptic reference orbit if the relative position vector is expressed in a local tangent coordinate system. The accuracy is probably more than adequate for many applications including manual rendezvous in earth orbit, up to an eccentricity of 0.05.

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A Linear Algorithm for Determining Relative Orbital State Using Angle Data

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THE linearized free-fall dynamics of a body slightly perturbed from a circular reference orbit, expressed in Cartesian local vertical coordinates of a body in the reference orbit, are¹

$$\frac{d}{d\tau} \begin{bmatrix} x \\ z \\ x' \\ z' \\ y \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \\ x' \\ z' \\ y \\ y' \end{bmatrix} \quad (1)$$

where $\tau = \omega t$, and $d(\)/d\tau = (\)'$. The homogeneous solution of these equations can be expressed in terms of the fundamental matrix $\Phi(\tau_2, \tau_1) \equiv \Phi_{21}$, as follows:

$$[xx'z'yy']_{\tau_2}^T = \Phi_{21}[xx'z'yy']_{\tau_1}^T \quad (2)$$

where

$\Phi_{21} =$

$$\begin{bmatrix} 1 & 6s_{21} - 6\tau_{21} & 4s_{21} - 3\tau_{21} & -2 + 2c_{21} & 0 & 0 \\ 0 & 4 - 3c_{21} & 2 - 2c_{21} & s_{21} & 0 & 0 \\ 0 & -6 + 6c_{21} & -3 + 4c_{21} & -2s_{21} & 0 & 0 \\ 0 & 3s_{21} & 2s_{21} & c_{21} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{21} & s_{21} \\ 0 & 0 & 0 & 0 & -s_{21} & c_{21} \end{bmatrix} = \begin{bmatrix} \Psi_{21} & 0 \\ 0 & \theta_{21} \end{bmatrix} \quad (3)$$

and where $\tau_{21} \equiv \tau_2 - \tau_1$, $c_{21} \equiv \cos \tau_{21}$, and $s_{21} \equiv \sin \tau_{21}$. The elements of the 4×4 Ψ_{21} matrix are denoted by Ψ_{ij21} ($i, j = 1, 2, 3, 4$), and the elements of the 2×2 θ_{21} matrix are denoted by θ_{ij21} ($i, j = 1, 2$). The elements of Φ_{21} are denoted by φ_{ij21} ($i, j = 1, \dots, 6$). In the case of an impulsively thrusting vehicle (or approximately for a finite-thrust orbit provided that the burn time is much shorter than one orbital period), the solution is modified by adding the velocity increments to x', y', z' at the time of their occurrence.

It has been shown that an unknown relative state $X_0 \equiv [x_0 z_0 x'_0 z'_0 y_0 y'_0]^T$ can be determined from a set of measurements of the in-plane angle δ and out-of-plane angle σ (Fig. 1), provided that an impulsive thrust maneuver of known ΔV -magnitude and direction is made by the perturbed body at some time during the measurement sequence.¹ References 1 and 2 define computational procedures for computing this state manually. This Note presents algorithms for performing the same task with a digital computer.

Denote the measurements by $\delta_1, \sigma_1, \delta_2, \sigma_2, \dots$ corresponding to times $\tau_1, \tau_2, \dots$ and states $x_1, z_1, x'_1, z'_1, \dots$. Denote the burn

made by the perturbed body at τ_b by $\Delta x', \Delta y', \Delta z'$, where τ_b is chosen at some time between the first and last measurement of the set, not too close to the bounds of the measurement interval to allow the change in velocity to propagate into position.

Two types of measurement-maneuver sequences are defined, the four-instant in-plane sequence, and the three-instant out-of-plane sequence. In the former, a set of four readings of δ and two of σ are taken, and the burn has only x and z components. In the latter, three pairs of δ, σ readings are taken, and the burn has a nonzero y component, the z and x components being either zero or nonzero.

The three-instant procedure is considered first. The state at time τ_i can be expressed

$$X_i = \Phi_{i0} X_0 + \Phi_{ib} V \quad (4)$$

where

$$V \equiv [000\Delta x'\Delta y'\Delta z']^T \quad (5)$$

$$\Phi_{ib} \equiv \begin{cases} [0] & \text{if } \tau_i < \tau_b \\ \Phi(\tau_i, \tau_b) & \text{if } \tau_i \geq \tau_b \end{cases} \quad (6)$$

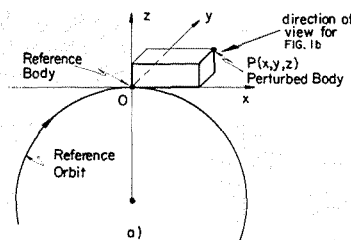
The individual state-components at time τ_i can be expressed

$$x_i = \Phi_{x i0} X_0 + \Phi_{x ib} V \quad (i = 1, 2, 3) \quad (7)$$

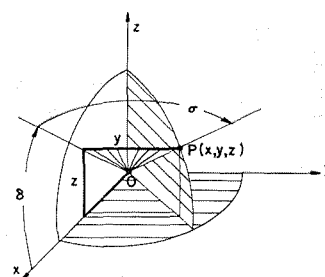
$$y_i = \Phi_{y i0} X_0 + \Phi_{y ib} V \quad (i = 1, 2, 3) \quad (8)$$

and a similar equation for z_i , where $\Phi_{x i0}$ denotes the 1×6 matrix whose elements are the top row of Φ_{i0} , $\Phi_{y i0}$ denotes the matrix whose elements are the fifth row of Φ_{i0} ; and $\Phi_{z i0}$ denotes the matrix whose elements are the second row of Φ_{i0} , and similarly for $\Phi_{x ib}$, $\Phi_{y ib}$, $\Phi_{z ib}$. Form the 3×1 matrix of x 's as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \Phi_{x10} \\ \Phi_{x20} \\ \Phi_{x30} \end{bmatrix} [X_0] + \begin{bmatrix} \Phi_{x1b} \\ \Phi_{x2b} \\ \Phi_{x3b} \end{bmatrix} [V] \quad (9)$$



Local Vertical Coordinate System



In-Plane & Out-of-Plane Angles

Fig. 1 Geometry of relative coordinate system showing measurement angles.

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This is abbreviated to matrix notation as

$$\mathfrak{X} = \bar{\Phi}_x X_0 + \bar{\Phi}_{xb} V \quad (10)$$

By similar steps and definitions, one obtains the vectors of the y and z components

$$\mathfrak{Y} = \bar{\Phi}_y X_0 + \bar{\Phi}_{yb} V, \quad \mathfrak{Z} = \bar{\Phi}_z X_0 + \bar{\Phi}_{zb} V \quad (11)$$

Denoting the tangent of the measurements of the δ 's as

$$m_i = \tan \delta_i = z_i/x_i \quad (i = 1, 2, 3) \quad (12)$$

one can write

$$\mathfrak{Z} = M\mathfrak{X} \quad M \equiv \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \quad (13)$$

From the geometry of Fig. 1,

$$\tan \sigma = y/(x^2 + z^2)^{1/2} \text{ and } \sin \delta = z/(x^2 + z^2)^{1/2} \quad (14)$$

Hence

$$y_i = z_i \tan \sigma_i / \sin \delta_i \equiv q_i z_i \quad (i = 1, 2, 3) \quad (15)$$

and one can write

$$\mathfrak{Y} = Q\mathfrak{Z} \quad Q \equiv \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \quad (16)$$

Inserting Eq. (10) and $\mathfrak{Z}$ from Eq. (11) into Eq. (13),

$$\bar{\Phi}_x X_0 + \bar{\Phi}_{xb} V = M\bar{\Phi}_x X_0 + M\bar{\Phi}_{xb} V \quad (17)$$

Transposing,

$$(\bar{\Phi}_x - M\bar{\Phi}_x)X_0 = (M\bar{\Phi}_{xb} - \bar{\Phi}_{xb})V \quad (18)$$

Similarly, inserting $\mathfrak{Y}$ and $\mathfrak{Z}$ from Eq. (11) into Eq. (16),

$$(\bar{\Phi}_y - Q\bar{\Phi}_y)X_0 = (Q\bar{\Phi}_{yb} - \bar{\Phi}_{yb})V \quad (19)$$

The 3×1 matrix equations in Eqs. (18) and (19) can be considered as partitions of the 6×1 equation:

$$AX_0 = BV \quad (20)$$

where

$$A \equiv \begin{bmatrix} \bar{\Phi}_x - M\bar{\Phi}_x \\ \bar{\Phi}_y - Q\bar{\Phi}_y \end{bmatrix} \quad B \equiv \begin{bmatrix} M\bar{\Phi}_{xb} - \bar{\Phi}_{xb} \\ Q\bar{\Phi}_{yb} - \bar{\Phi}_{yb} \end{bmatrix} \quad (21)$$

The solution of Eq. (20) yields the desired result,

$$X_0 = A^{-1}BV \quad (22)$$

Equation (22) is the only equation that need be solved by the computer, given the definitions of A , B , V , and the data. Once X_0 is known, state at any other time can be found using Eq. (4).

Using the subscript p to distinguish the four-instant in-plane solution, define the following symbols:

$$X_p \equiv [xx'z']^T \quad (23)$$

$$\mathfrak{X}_p \equiv [x_1 x_2 x_3 x_4]^T \quad (24)$$

$$\mathfrak{Z}_p \equiv [z_1 z_2 z_3 z_4]^T \quad (25)$$

$$M_p \equiv \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \quad (26)$$

$$V_p \equiv [00\Delta x'\Delta z']^T \quad (27)$$

$$\Psi_{ib} \equiv \begin{cases} [0] & \text{if } \tau_i < \tau_b \\ \Psi(\tau_i, \tau_b) & \text{if } \tau_i \geq \tau_b \end{cases} \quad (28)$$

The in-plane state X_{ip} at any time τ_i can be expressed in

terms of initial conditions as follows:

$$X_{ip} = \Psi_{i0}X_{0p} + \Psi_{ib}V_p \quad (i = 1, 2, 3, 4) \quad (29)$$

The individual components of position at τ_i are

$$x_i = \Psi_{x i0}X_{0p} + \Psi_{x ib}V_p \quad (i = 1, 2, 3, 4) \quad (30)$$

$$z_i = \Psi_{z i0}X_{0p} + \Psi_{z ib}V_p \quad (i = 1, 2, 3, 4) \quad (31)$$

where $\Psi_{x i0}$ and $\Psi_{z i0}$ are the first and second rows respectively of Ψ_{i0} . Combining all four values of x into a matrix,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \Psi_{x10} \\ \Psi_{x20} \\ \Psi_{x30} \\ \Psi_{x40} \end{bmatrix} [X_{0p}] + \begin{bmatrix} \Psi_{x1b} \\ \Psi_{x2b} \\ \Psi_{x3b} \\ \Psi_{x4b} \end{bmatrix} [V_p] \quad (32)$$

which is abbreviated, using prior conventions, as

$$\mathfrak{X}_p = \bar{\Psi}_x X_{0p} + \bar{\Psi}_{xb} V_p \quad (33)$$

and a similar equation applies for the z -components. Using Eqs. (26) and (12), one can write

$$\mathfrak{Z}_p = M_p \mathfrak{X}_p \quad (34)$$

Substituting $\mathfrak{Z}_p$ and $\mathfrak{X}_p$, and transposing,

$$(\bar{\Psi}_x - M_p \bar{\Psi}_x)X_{0p} = (M_p \bar{\Psi}_{xb} - \bar{\Psi}_{xb})V_p \quad (35)$$

Solving for X_{0p} yields the results for the in-plane state

$$X_{0p} = (\bar{\Psi}_x - M_p \bar{\Psi}_x)^{-1} (M_p \bar{\Psi}_{xb} - \bar{\Psi}_{xb})V_p \quad (36)$$

Knowing the in-plane state at τ_0 , the in-plane state at any other time can be found using Eq. (29).

The out-of-plane state can be found by obtaining the values y_i, y_k using Eq. (15) for any two measurement times between which there was no y component of thrust;

$$y_i = q_i z_i, \quad y_k = q_k z_k \quad (37)$$

By virtue of the nonthrust, it is clear from Eq. (3) that $y_k = c_{ki}y_i + s_{ki}y_i'$, which is solved for y_i'

$$y_i' = (y_k - c_{ki}y_i)/s_{ki} \quad (38)$$

Equations (36–38) are implemented in the computer, completing the determination of state at τ_i in the four-instant procedure. Knowing the complete state at τ_i , the state at any other time is found using Eq. (4), identifying 0 with j .

When an out-of-plane thrust maneuver is contemplated, the three-instant out-of-plane procedure should be used. When an in-plane thrust maneuver is contemplated, the four-instant in-plane procedure will ordinarily give the most accurate results. Whereas in principle the three-instant procedure can be used with any finite burn, in the case in which burn has no y component this procedure will give accurate results only when the out-of-plane distance y is comparable with the in-plane distance $(x^2 + z^2)^{1/2}$.

The 3-instant out-of-plane procedure using Eq. (22) requires inversion of a 6×6 matrix. The 4-instant in-plane procedure using Eq. (36) requires inversion of a 4×4 matrix. Obviously the solution can be implemented in other ways, since both were solved in the manual method of Ref. 1 by inverting nothing larger than 2×2 matrices.

There are occasions when one or the other of the above methods will fail, this being evidenced by the difficulty in inverting the matrix. Under these circumstances, the difficulty is usually corrected by using an extra measurement of δ and σ in place of one δ - σ pair from the original set.

The algorithms presented here can be extended to the case of moderately eccentric reference orbits using the techniques of Ref. 3, in which the x, y, z axes are simply reinterpreted as defined by a local tangent coordinate system, there being no change to the equations of motion or their solution.

The algorithms presented here can also be extended to the case where separation distances along the orbit are longer than

are accurately described by the linear equations (1) by using the technique of Ref. 4. There, curvilinear axes are introduced in which the equations of motion and their solution again have exactly the same form as given here.

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SIMULATION AND ERROR ANALYSIS
of a
MANUAL RENDEZVOUS SYSTEM

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ABSTRACT

A system for navigation, guidance, and control of a spacecraft to rendezvous with an orbiting target, based entirely on observations by handheld, unpowered instruments, and computations done entirely by hand, has been developed. This paper describes results of an interactive simulation of this system through a selected set of rendezvous missions, demonstrating feasibility in the presence of realistic errors. The simulation is carried out by a trained operator, the "astronaut," working in conversational mode with a desk-size digital computer. The computer represents the environment, and the operator represents the astronaut's decision-making process. Plots are given showing the increase in fuel and in a composite fuel-time cost as a function of the magnitude of the principal error sources, namely: model error due to separation range, model error due to eccentricity of the reference orbit, control errors in direction and magnitude of burn vectors, and measurement error. Results of a manual computation error study are also presented. It is shown that the manual rendezvous system is strongly convergent, so that the real penalty of errors is not failure to rendezvous, but rather, using more fuel and/or time than would otherwise have been necessary.

INTRODUCTION

A system for navigation, guidance, and control of a spacecraft to rendezvous with an orbiting target, based entirely on observations by handheld, unpowered instruments, and computations done entirely by hand, has been developed.⁽¹⁾ This system uses an optical sextant developed by the Air Force for use in space.⁽²⁾ The system was designed in cooperation with the developers of the sextant, and was made to be compatible with a scheme they had worked out for orbit determination.⁽³⁾ A prototype model of the sextant was flown and space-tested in the Gemini IV and VII flights, the results showing it to possess more

than sufficient accuracy for the intended purpose for which it was developed.⁽⁴⁾

This present paper describes results of an interactive simulation of the manual guidance system through a selected set of rendezvous missions, together with results of studies of the principal sources of error.*

The simulation is carried out by a trained operator, the "astronaut," working in a conversational mode with a desk-size digital computer. The computer keeps track of the exact position of target and spacecraft relative to inertial space, and to each other. It implements spacecraft maneuvers called for by the astronaut, suitably changing the orbital parameters of the spacecraft. It keeps track of light and dark periods, assuming the target to be visible only in the dark for purposes of sextant observations. It maintains the "star field," i.e. celestial coordinates, and gives the astronaut the sighting angles that would be observed between the line-of-sight to the target and the line-of-sight to selected stars.

The astronaut makes the human decisions. He decides the number of sightings to be made and the time at which each is taken. By interpreting the results of his sightings according to the manual guidance system, he decides what kind of flight plan to implement -- the direction, magnitude, and time of each burn -- and then "commands" each burn by notifying the computer of his decision. Working close to real time, he leads the computer through the complete rendezvous mission. In short, the computer represents the environment, and the operator represents the astronaut's decision-making processes.

For the reader who does not have immediate access to Ref. 1, Appendix A summarizes the essential features of the manual rendezvous system.

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Sources of error include: linearized model error due to finite separation range; model error due to eccentricity of the reference orbit; control errors in direction and magnitude of burn vectors; measurement error including sextant, window, and operator contributions; computation (round-off) error; errors in the method of star sight averaging (made to compensate for non-simultaneous measurements); curvature error (model error due to the use of Cartesian rather than curvilinear coordinates); finite burn time (the model assumes impulses); variable thrust acceleration (the model assumes constant thrust acceleration); gravitational field due to oblate rather than spherical earth; errors in knowledge of period and plane of reference orbit; etc. It will be shown that the real penalty of errors is not failure to rendezvous, but rather, greater use of fuel and/or time than would otherwise have been necessary.

This study seeks to evaluate the manual system through a numerical analysis of what are believed to be the principal error sources, namely, the first five of the preceding list. Data on each of the first four error sources is obtained from digital simulations of a series of complete rendezvous missions in which only a single error source is allowed to act at an increasing level of severity. The fifth source is evaluated by comparing results of manual and digital computations. Then, selected missions are carried out with anticipated realistic values of several error sources acting concurrently. The evaluation is felt to be reasonably representative of what should be obtainable in practice.

SIMULATION SET-UP

Digital simulation of the manual rendezvous scheme is performed on an IBM 1130 computer, a desk-size machine featuring internal disk storage. A standard card input is used to load the program, and a console consisting of a typewriter keyboard and a few special keys for controlling computer functions serves as the astronaut's input device. Output is obtained from the console printer using a typing ball to print the characters at rapid speed. Internal computations are done double precision giving results to 16 decimal-digits.

This computer was chosen because 1) its dollar cost per hour is low enough to be feasible for completely dedicated use by a human operator whose reaction time is significant, and 2) its input-output lends itself to the kind of operation desired, i. e. insertion of an astronaut decision, computation, output of a computational result reflecting a change in the environment, evaluation of the result by the astronaut, and finally, a new astronaut decision, repeating the process. The

structure of the FORTRAN program used in the simulation is designed to implement this conversational mode. The program is best described as consisting of a main inter-communication section which interfaces to more than 20 subroutines for the basic calculations. As an illustration, at any point in the rendezvous mission, the astronaut decides the next operation he wishes to perform. He requests an interface by typing a one or two letter code, which tells the computer which subroutine will be used, and enters the necessary input data. The subroutine performs the calculations and returns the results to the main program which communicates the information to the astronaut via the console printer. Based upon these new results, the astronaut will then make his next decision and issue a request for the same or a new interface.

In short, the computer is only a tool which the operator uses to simplify his tasks. In the simulation, as in the actual mission, the astronaut must achieve the rendezvous.

A. Computer Functions

The computer program is divided into three major groups of functions. The subroutines shown in Table 1 simulate the environment, or "real world." Initial conditions are specified to the program by inserting the state vector (position and velocity) of each vehicle in absolute coordinates at the time of target orbit perigee passage. Time is arbitrarily set to 0 hours at this instant. Given these initial state vectors, the real world portion of the program propagates the two orbits in inertial space over a spherical earth with inverse-square force field using the Keplerian equations of motion. The measurement process is simulated by the operator's specifying the time at which a real astronaut would actually use his sextant. The computer updates the position of both vehicles relative to the stars to the selected time. It then has the information to generate the angles which a sextant would read at that time. As a time-saver, instead of calculating the direct sextant angles from such a measurement, the program goes one step further and computes the line-of-sight angles σ and δ as seen in the LTCS. (For definitions of σ , δ and LTCS, see Appendix A.) This automatic data reduction avoids the necessity of the astronaut having to do a spherical triangle calculation (transformation from celestial coordinates to LTCS). Since it is assumed for this simulation that the astronaut can only take sextant measurements when the target is in the dark (this is probably overly conservative), the real world simulation keeps track of the relation of the target vehicle to the light and dark portions of its orbit.

TABLE 1

Computer Program Subroutines (Real World)	
Name	Calculation
SIGMA	Line-of-sight angles in LTCS.
TRUST	Projects an absolute state vector over a given time interval.
RELST	Orientation of LTCS unit vectors in inertial space at any time.
RANGE	Length of relative state vector in inertial space.
BURN	Velocity change to interceptor upon thruster firing.
DK	Table of dark intervals.
MAIN PROGRAM	Total fuel (ΔV) used in maneuvering astronaut's vehicle.

The second major group of functions performed by the computer are listed by subroutines in Table 2. These subroutines carry out the "manual computations," i. e. those which the astronaut would normally perform using the charts, tables, and graphical procedures outlined in Ref. (1). The reason they have been computerized is to save the operator's time in conducting simulated missions. In totality, they are functionally equivalent to the manual computations, producing the same outputs for a given set of inputs.

The third group of computer functions is one of bookkeeping, and simplifies the task of evaluating simulation results. For example, the computer keeps track of the true relative state vector to use as a reference against which the astronaut's estimate can be compared, and errors determined. Careful distinction is always made between the data legitimately available to the astronaut and data available only to the operator in his role as designer and evaluator of the system.

B. Operator's Functions

In the role of astronaut, the operator simulates all of the human decisions which would be made in a real mission regarding the navigation, guidance and control of his vehicle in relation to the target. He decides 1) the time of sextant readings, 2) the magnitude, direction and time of all burns, 3) the method of state determination, 4) the type of flight plans to be used, and 5) the con-

TABLE 2

Computer Program Subroutines (Astronaut's World)	
Name	Calculation
INPLN	Determine in-plane normalized state from sightings.
STATE	Project state over a specified time interval using linearized equations.
PHASE	Determine plane-crossing time.
OOP3	Find dimensional state with non-optimal out-of-plane burn.
OPDIM	Find dimensional state with optimal out-of-plane burn.
FORIP	Determine state by 4-instant-in-plane method.
TWOBN	Obtain an immediate 2-burn flight plan.
TBMAN	Obtain a 3-burn flight plan.
WABRN	Obtain a wait-and-2-burn flight plan.

ditions under which the rendezvous mission is terminated. In making these decisions, he normally follows the procedures of the manual system, using the data obtained in his previous calculations. However, he is entirely free at any time to disregard the standard procedures and rely on his physical intuition and the insight gained from experience in using the manual rendezvous technique. Although the decision-making time used by the operator in a mission is nearly equal to that of an astronaut, computerization of all calculations and instantaneous update of trajectories over specified time intervals allows the former to achieve rendezvous in $1/4 - 1/3$ the time it would actually take in real life.

DESCRIPTION OF TEST CASES

For the purposes of this study, three test cases were selected. Since the manual rendezvous method is perfectly general and capable of rendezvous from any initial condition (provided only that the target remains visible until the first state-determination) the cases were chosen to cover a wide span among the total range of possibilities. Two methods may be used to completely define initial conditions for one case: 1) the initial relative state vector in the LTCS, or 2) the parameters r , $\bar{r}$, the initial value δ_0 , and elapsed

time to the first crossing of the target-interceptor orbital plane, PC. In either case, it is also necessary to specify the time at which the first dark period begins, DB, the reference orbit eccentricity, the period of the reference orbit, T , and the cost-of-time weighting constant, k . Upon suitable reduction of the data, the methods 1) and 2) above are equivalent.

The three test cases are defined in Table 3. For reasons to be discussed in the next section, the relative state vectors in the table have been normalized to a separation range** of .2 nautical miles (nm). Figure 1 shows the true relative trajectory in the LTCS for each case in the absence of thrusting.

TABLE 3

Test Cases				
	Case 1	Case 2	Case 3	Units
x_0	-.1999	.2	.126	nm
y_0	.0095	.028	.085	nm
z_0	-.01	.02	-.126	nm
$\dot{x}_0$	.061	-.144	.256	knots
$\dot{y}_0$	.0123	.08	-.1248	knots
$\dot{z}_0$	.0	.16	.382	knots
ϵ	.0	.0	.0	
r	.9	-.2	.17	
$\bar{T}_0$	.0	1.65	1.70	rad
δ_0	(.05- π)	.1	$-\pi/4$	rad
PC	.3T	.35T	.19T	
DB	.0	.0	.2	hrs
T	1.5697	1.5697	1.5697	hrs
k	.0	.0217	.0608	knots/rad

Case 1 is typical of the situation in which the interceptor vehicle is launched to rendezvous with a target whose orbit is reasonably well known. It is placed in good position for rendezvous, at a lower altitude, and slightly behind the target. The fact that the interceptor orbit is nearly, but not quite, coelliptical (in which case r would be 1 instead of 0.9) is a reflection of an assumed slight uncertainty in the target orbit parameters by the system that established the interceptor in its present situation.

* The orbital period of one vehicle or the other must be known by an astronaut desiring to use the manual method. This can also be found by manual means. (3)

** Separation Range = $\sqrt{x^2 + y^2 + z^2}$

Case 2 was chosen to illustrate the capability of the method for achieving rendezvous when the astronaut's altitude before maneuvering was sometimes higher and sometimes lower than the target vehicle, reflected in the fact that r is negative. In addition, the relation between the orbital plane-crossing times and the in-plane extremum times ("apogee" and "perigee" of the relative trajectory) represents a more difficult condition than in Case 1.

Case 3 was designed to test the full capability of the method under extremely (and perhaps unrealistically) difficult conditions. The magnitudes for each component of the initial relative state vector were chosen at random. A series of numbers from 0-10 inclusive were written on equal size slips of paper, and one blindfolded member of the study group drew one slip at a time from a hat. The chosen slip was replaced in the hat before the next selection was made. A coin flip was used to determine the sign of each component.

STRATEGIES FOR THE TEST CASES

Before evaluating the manual rendezvous method in light of its principal error sources, it is necessary to define pragmatically what constitutes an "error-free" mission on the digital simulator. The criteria of Table 4 were used throughout the study.

The data given in Table 3 establishes initial conditions for error-free trajectories, with respect to both separation range and reference orbit eccentricity.

In the remaining sections of this paper, the particular rendezvous mission of each test case which satisfies the rules of Table 4 will be known as the reference mission of the case. The test plan calls for evaluating the principal error sources one by one. A given error-producing effect is introduced into a rendezvous mission that has all other error sources held at the reference mission values, and a new rendezvous mission is run with this error source. A series of such runs is made, with the error source set successively at higher values. The successive missions related to a given reference mission will be denoted by the magnitude of the individual error source present -- e.g. the 10-mile range mission of case 1, the .03 eccentricity mission of case 3, etc.

To achieve rendezvous from a given set of initial conditions requires the implementation of a strategy -- the particular sequence of measurements and the kind(s) of flight plans used by the astronaut during a mission. In principle, it is possible to determine an optimum strategy for each mission separately. However, in this study, in order to

evaluate the influence of each individual error source, a common strategy was normally used for all missions of a given test case regardless of the magnitude or type of error present. Several strategies for a given case were investigated before one was selected which would represent a reasonable approximation to an optimal strategy for all related missions for a given case. The use of one common strategy also makes it possible to compare the error sources to determine their relative influence on the manual rendezvous scheme.

TABLE 4

Standard Conditions for "Error-Free" Missions	
Error Source	Magnitude
Separation Range	.2 nm at initial time -- this short range effectively nulls the model error in the linearized equations due to finite range.
Eccentricity of Reference Orbit	The target orbit is circular ($e = 0$) with radius being 3700 nm -- this nulls the model error in the linearized equations due to eccentricity.
Control Errors in Burn Vector	Magnitude - accurate to nearest .00005 knot*. Direction - accurate to nearest .00005 rad.*
Measurement Error	Line-of-sight angles σ and δ accurate to nearest .00005 rad.*
Computation Error	All computations normally done using charts and tables entered accurately to nearest .00005 units.

* These values establish the level of round-off error of the operator in entering data.

This comparison would be less meaningful if different strategies were used for successive missions of a given case. Tables 6, 7 and 8 present the astronaut's reasoning and the resulting strategy for each case. The operator considers himself rendezvoused with the target if both

Final Separation Range $\leq$.01 x (Maximum Separation Range)

Final Velocity Magnitude (knots) $\leq$ 4 x (numerical value of Final Separation Range in nm)

Figures 2-4 show the use of the strategy on the reference mission of each test case. The symbols for major events used in these figures and in Tables 6-8 are given in Table 5.

TABLE 5

Symbols for Events	
Symbol	Event Description
⊙	Measurement
⊙→	In-plane thrust maneuver (arrow indicates direction)
⊗	Out-of-plane thrust maneuver.
3	3-instant out-of-plane state determination.
4	4-instant in-plane state determination.
DB	Dark period begins
DE	Dark period ends
2B	2-burn maneuver calculation
3B	3-burn maneuver calculation
SP	State Projection
Time Definition	
A _p	Apogee passage
P _e	Perigee passage
PC	Plane-crossing

Case 1 (see Fig. 2)

The interceptor is in a nearly coelliptic orbit, behind, and slightly below the target. Rather than attempting to rendezvous in minimum time, the astronaut decides to minimize fuel only. Hence, $k = 0$ for this case. He decides to adopt a policy of coellipticity* that will ensure a sequence of very good state determinations throughout the mission. After in-plane coasting for 1-1/2 orbits, a two-burn maneuver places his vehicle in a coelliptic parking orbit with a small differential altitude. From this situation, an accurate state determination is made verifying his new orbit. A simple Hohmann transfer completes the mission.

* Coelliptic orbits are not specifically included in the present formulation of the Flight Planning Charts of the manual method.

TABLE 6

Strategy - Case 1			
	Event	Time (Hrs.)	Reasoning
1.	DB	.0	Dark begins.
2.	☉	.0001 .2 .4	These measurements will determine the times of astronaut orbit perigee-apogee passage and the orbital plane-crossing time.
3.	☉	PC*	Calibrated thrust removing approximately 1/2 remaining out-of-plane velocity magnitude. Residual velocity is purposely left to aid in future state determinations.
4.	☉	.6	A measurement after the out-of-plane thrust is made to set up state determination using [3].
5.	[3]		Measurements at .2, .4, .6 and out-of-plane thrust are used to determine state at .4 hours.
6.	SP		State is projected to the apogee passage time where a possible in-plane thrust could be made. The astronaut sees he is making good progress and decides to coast for at least one full orbit.
7.	☉	.78	Measurement taken just before leaving the dark. It is good practice to take extra measurements in case a previous measurement should turn out to be faulty.
8.	DE	.7848	Dark ends.
9.	SP	1.0 1.2 1.4	The known state at .4 hrs is projected forward to monitor progress during the light period.
10.	DB	1.5698	

TABLE 6
(Continued)

Strategy - Case 1			
	Event	Time (Hrs.)	Reasoning
11.	☉	1.57 1.76 1.95	See Event 2.
12.	☉	PC	See Event 3.
13.	☉	2.15	See Event 4.
14.	[3]		Measurements at 1.76, 1.95, 2.15 and out-of-plane thrust are used to redetermine state at 1.95 hrs. Since the separation range is reduced, this determination should be more accurate than the one at Event 5.
15.	SP		State is projected to apogee passage time. Astronaut sees that waiting one more orbit would put him ahead of target, an undesirable situation. He decides to initiate a 2-burn maneuver putting him into a coelliptic orbit with smaller altitude differential from target.
16.	☉	A _p	First burn of maneuver.
17.	☉	2.35	See Event 7.
18.	DE	2.3546	
19.	SP	2.55 2.75 2.95	See Event 9 (known state at 1.95 hrs).
20.	☉		Second burn of maneuver. This burn should establish the interceptor in a coelliptical ($r = 1$) orbit.
21.	DB	3.1394	
22.	☉	3.14 3.33 3.52	See Event 2.

*The time of this maneuver is the astronaut-calculated plane-crossing time, which differs slightly from one mission to the next.

TABLE 6
(Continued)

Strategy - Case 1			
	Event	Time (Hrs)	Reasoning
23.	⊗	PC	See Event 3.
24.	⊙	3.75	See Event 4.
25.	[3]		Measurements at 3.33, 3.52, 3.75 and out-of-plane thrust are used to redetermine state at 3.52 hrs. The [4] procedure using the burn entering the parking orbit could have been used but there was sufficient time for the more accurate [3] procedure.
26.	SP		State at 3.52 hrs is projected to the time of the first burn of Hohmann transfer.
27.	⊙	3.92	See Event 7.
28.	DE	3.9242	
29.	2B		Calculations of the burn vector components for the Hohmann transfer.
30.	⊙		The first burn of a two-burn transfer is made when astronaut's estimated state crosses the Hohmann line*.
31.	SP	4.15 4.35 4.55	See Event 9 (known state at 3.52 hrs).
32.	⊗	PC	All remaining out-of-plane velocity is removed at this plane crossing time which was determined from the previous state. The astronaut assumes he will not need another state determination.
33.	DB	4.7090	
34.	⊙		Second burn of transfer at 1/2 orbit after previous burn. Interceptor is now in favorable position for docking.

*The line from which a Hohmann transfer causes an interceptor in a coelliptic orbit to meet the target.

Case 2 (see Fig. 3)

Rendezvous in this case can be achieved after one state determination by employing a 3-burn maneuver sequence. The first two burns of the sequence are made 1/2 reference orbit apart and place the interceptor into the proper phasing orbit for rendezvous 1/2 period later. However, for missions in which an error source is present, the phasing orbit will generally not meet the target at the scheduled instant of the final burn. This will be due to deviations between the true trajectory and the astronaut's estimate. To alleviate this problem, the astronaut re-determines state while in the phasing orbit, and calculates an immediate two-burn maneuver to end approximately at the time of the original third burn. The two-burn maneuver will null any errors which have accumulated. It should be noted that only two state-determinations are made, compared to three in case 1.

TABLE 7

Strategy - Case 2			
	Event	Time (Hrs)	Reasoning
1.	DB	.0	
2.	⊙	.0001 .2 .4	See Event 2 - Case 1
3.	⊗	PC	See Event 3 - Case 1
4.	⊙	.6	See Event 4 - Case 1
5.	[3]		See Event 5 - Case 1
6.	SP		State is projected to the perigee passage time where a possible in-plane thrust could be made. The astronaut sees that he is in a favorable position for a three-burn rendezvous maneuver and decides to initiate this flight plan.
7.	3B		Calculations of the components of each burn vector of a three-burn flight plan are made. The first burn raises apogee, the second (one-half period later) is intended to raise perigee to 0, and the third (one-half period after second) will complete rendezvous, if state and maneuvers are perfect.

TABLE 7
(Continued)

Strategy - Case 2		
Event	Time (Hrs)	Reasoning
8. $\odot$	.78	See Event 7 - Case 1.
9. DE	.7848	
10. $\odot$	P_e	First burn of maneuver.
11. SP	1.0 1.2 1.4	See Event 9 - Case 1.
NOTE: The astronaut suspects that, because of normal accumulation of error, there may be some inaccuracy in his state determination used in calculating the 3-burn flight plan. He decides to forego the third burn, and instead to obtain a new, more accurate state determination at close range, terminating the mission with an immediate 2-burn maneuver.		
12. DB	1.5698	
13. $\odot$	1.57	Since the second burn will occur in the dark, there is insufficient time afterward for state determination using [3]. Instead the in-plane burn will be used for a determination using [4].
14. $\odot$		Second burn of maneuver at 1/2 reference orbit after first burn.
15. $\odot$	1.80 1.95	These are the second and third measurements needed for using [4].
16. $\odot$	PC	Since no further state determinations will be required, the remaining out-of-plane velocity is eliminated. This in no way affects the present state determination process.
17. $\odot$	2.25	Fourth measurement for [4].
18. [4]		Measurements at 1.57, 1.80, 1.95, 2.25, and the second in-plane burn are used to redetermine state at 2.25 hrs.

TABLE 7
(Continued)

Strategy - Case 2		
Event	Time (Hrs)	Reasoning
19. SP		State is projected to the time of the original third burn. The astronaut predicts the target miss, resulting from his present trajectory.
20. 2B		To correct predicted miss, he sets up new two-burn flight plan; this is his terminal guidance.
21. DE	2.3546	
22. $\odot$		First burn of maneuver.
23. $\odot$		Second burn of maneuver. Upon completion of this burn, the astronaut is in position for docking. Target can be observed visually since this burn occurs in the light.

Case 3 (see Fig. 4)

The randomly chosen test case presents the astronaut with among the most difficult set of initial conditions from which to achieve rendezvous. The astronaut's vehicle is below and forward of the target, and moving away, causing the separation range to markedly increase in a short period of time. Under such conditions, rapid state determination and maneuvering is essential so that the target will not be lost. However, although he may be aware of his predicament by observing the receding target, no readings can be taken for .2 hrs until the dark period begins. Ordinarily, for fuel optimality, one chooses to wait to make an out-of-plane calibrated thrust maneuver at a plane-crossing. In this case, because of the urgency for determining state quickly, fuel-optimality is sacrificed, and an out-of-plane burn is made at the earliest time consistent with the [3] procedure, which the astronaut then uses to determine state. The first two burns of a three-burn flight plan establish him at a higher average altitude, so that he is closing on the target. From this condition, he elects to coast until the separation range is reduced and state is accurately redetermined, whereupon he employs a flight plan quite similar to that discussed in case 2.

It is a simple matter to remove any remaining out-of-plane velocity once the plane-crossing times have been accurately determined.

TABLE 8

Strategy - Case 3		
Event	Time (Hrs)	Reasoning
1. DB	.2	
2. ⊗	.2001	First measurement for use in [3] procedure.
3. ⊗	.3	Astronaut chooses a time instant and makes a calibrated thrust of approximately 1/2 his remaining out-of-plane velocity. Had normal procedures been followed, a calculated plane-crossing time of 1.08 hrs would have resulted.
4. ⊗	.4 .6	Second and third measurements needed for using [3]. The in-plane extremum times have now been calculated.
5. [3]		Measurements at .2001, .4, .6 and estimated thrust are used to determine state at .6 hrs. Note: [3] can be used even though the burn is between measurement 1 and 2 instead of 2 and 3 as in the other test cases.
6. SP		The known state is projected to the perigee passage time and the astronaut quickly decides to use a 3-burn maneuver.
7. 3B		See Event 7 - Case 2.
8. ⊖	P _e	First burn of maneuver.
9. ⊗	.98	See Event 7 - Case 1.
10. DE	.9848	
11. SP	1.00 1.35 1.6	The known state at .6 hrs is projected forward to monitor progress during the light period.

TABLE 8
(Continued)

Strategy - Case 3		
Event	Time (Hrs)	Reasoning
12. ⊖		Second burn of maneuver at 1/2 reference orbit after first burn.
13. DB	1.7696	
14. ⊗	1.77	See Event 2 - Case 3.
15. ⊗	PC	A calibrated thrust is made at the plane-crossing time determined from the previous state determination. One-half of the remaining out-of-plane velocity is removed, the residual being left for future state determinations.
16. ⊗	2.0 2.3	See Event 4 - Case 3.
17. [3]		Measurements at 1.77, 2.0, 2.3 and calibrated thrust used to determine state at 2.3 hrs. The astronaut sees he is satisfactorily closing on the target and decides to monitor his progress for at least one more orbit before applying midcourse corrections.
18. ⊗	2.55	See Event 7 - Case 1.
19. DE	2.5544	
20. SP	2.9 3.15	See Event 11 - Case 3 (known state at 2.3 hrs).
21. DB	3.3392	
22-26		The events during this dark period are identical to Events 14-18. Measurements are taken at 3.34, 3.6, 3.85 hrs with state being redetermined at the latter time. All reasoning is identical, as well.
27. DE	4.1240	
28. SP	4.35 4.6	See Event 11 - Case 3 (known state at 3.85 hrs).

TABLE 8
(Continued)

Strategy - Case 3		
Event	Time (Hrs)	Reasoning
29. DB	4.9088	
30. ☉	4.91	See Event 2 - Case 3.
31. ☉	PC	All of the remaining out-of-plane velocity is removed with this burn.
32. ☉	5.15 5.4	See Event 4 - Case 3.
33. [3]		Measurements at 4.91, 5.15, 5.4 are used to redetermine state at 5.4 hrs.
34. SP		State is projected to the perigee passage time. A perfect trajectory would rendezvous at this time. If a target miss occurs, then midcourse guidance (corrective maneuvers) is needed. By referring to the Flight Planning Charts, the astronaut decides to implement a 3-burn flight plan to rendezvous.
35. 3B		See Event 7 - Case 2*
36. ☉	P _e	First burn of midcourse maneuver.
37. ☉	5.69	See Event 7 - Case 1.
38. DE	5.6936	
39. SP	6.0 6.3	See Event 11 - Case 3 (known state at 5.4 hrs).
40. ☉		Second burn of midcourse maneuver at 1/2 reference orbit from first burn.
41. DB	6.4784	
42. ☉	6.5 6.7 6.9	The astronaut decides to use the second burn (Event 40) for a state determination using [4].

* The first burn of this three-burn maneuver will lower apogee.

TABLE 8
(Continued)

Strategy - Case 3		
Event	Time (Hrs)	Reasoning
43. [4]		Measurements at 5.69, 6.5, 6.7, 6.9 and the second burn are used to redetermine state. This new state determination will be more accurate than a state projection from previous state because data is more recent and range is shorter. Note: This method applies even though the sightings are split over two dark periods.
44. SP		State is projected to planned time of rendezvous to determine target miss, if any. If there is a miss, then terminal guidance (corrective maneuvers) is required.
45. 2B		Calculation of the components for each burn of a two-burn maneuver which will reduce remaining errors below the rendezvous tolerance (terminal guidance).
46. ☉	7.1	First burn of maneuver.
47. ☉		Second burn of maneuver. Upon completion of this burn, the astronaut is in position for docking.

PROCEDURE

With the magnitudes of the principal error sources set at the "error-free" conditions of Table 4, the complete reference mission for each test case was simulated and data was obtained on both the total fuel expenditure, as measured by ΔV , and time in achieving rendezvous. This data, together with cost, appears in Table 9. Unless otherwise specified, ΔV is expressed in knots, range in nm, and time in hours (hrs) throughout.

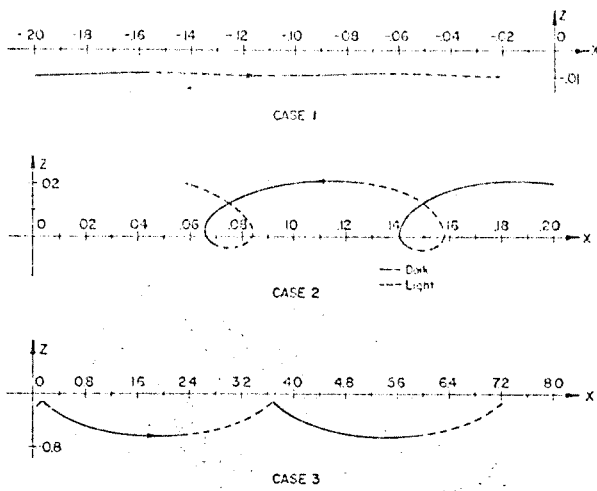


FIG. 1. TEST CASE INITIAL CONDITIONS.

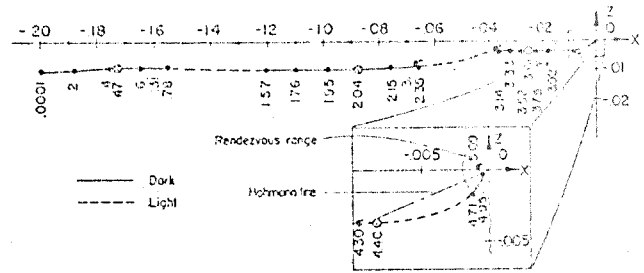


FIG. 2. REFERENCE TRAJECTORY - CASE 1

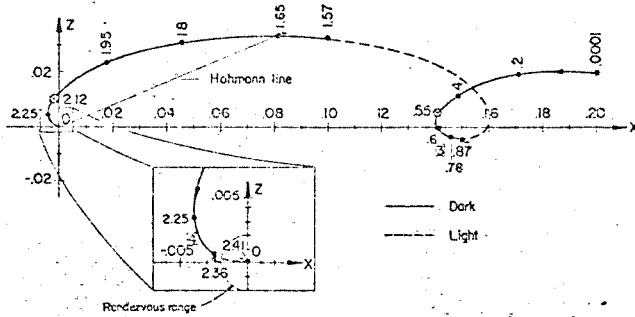


FIG. 3. REFERENCE TRAJECTORY - CASE 2

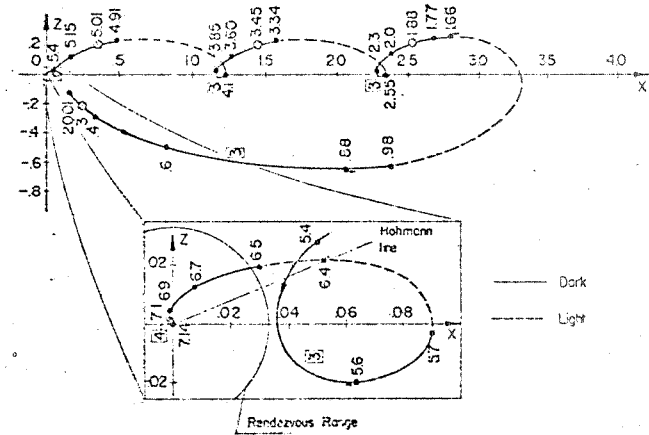


FIG. 4. REFERENCE TRAJECTORY - CASE 3

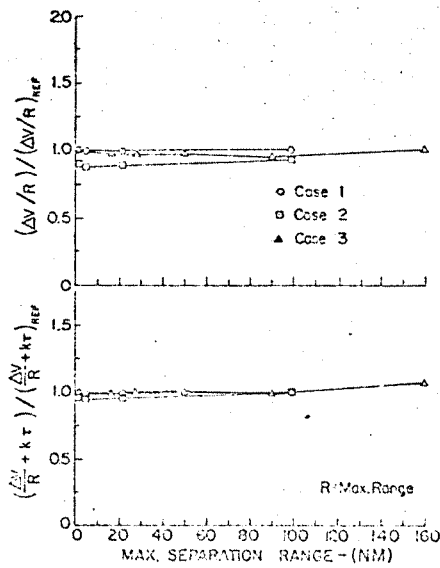


FIG. 5. MISSION PERFORMANCE RELATIVE TO SEPARATION RANGE

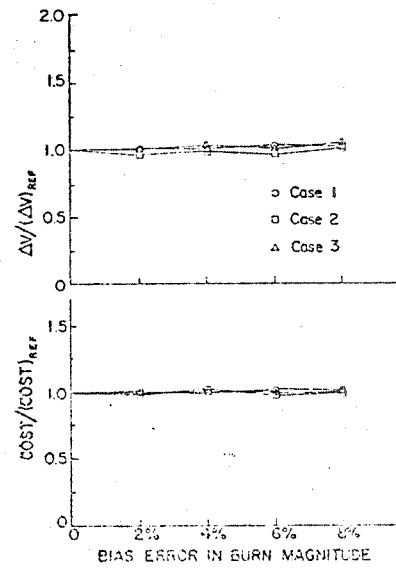


FIG. 6. MISSION PERFORMANCE RELATIVE TO BIAS IN BURN MAGNITUDE

TABLE 9

Results of Reference Missions			
	Case 1	Case 2	Case 3
Total ΔV (knots)	.0594	.2144	1.7002
Rendezvous Time (hrs)	5.09	2.41	7.14
Cost (knots)	.0594	.423	3.39

Then, a given error-producing effect was introduced, each test case re-run with all other error sources held at the reference mission values, and new results on fuel and time obtained. A series of such rendezvous missions was simulated with the magnitude of the error source of interest set at successively higher values. Any velocity remaining when the rendezvous criterion is met is added, using the "cost of a trajectory" formula⁽¹⁾, to the ΔV spent in burns, to arrive at the total ΔV for the mission. The excess ΔV or cost required in a case with error present is an indication of the mission penalty which such an error imposes.

Model error due to separation range was introduced by scaling the initial relative state vector of each test case. If there were no model error, the only effect upon the reference mission data would be an increase in fuel in proportion to the scale factor. The time to achieve rendezvous would be constant since at any instant the state vector of the separation range mission would be directly proportional to that of the reference mission. However, since there is model error, state determinations using the linearized equations would be expected to yield increasingly imprecise values as the separation range is increased. The resulting deviations between the true trajectory and the astronaut's estimate may cause an increase in both fuel and time over the previously mentioned values.

Model error due to eccentricity of the reference orbit was introduced by placing the target in an absolute orbit of finite eccentricity, while keeping its period and the initial relative state at the reference mission values. Since the linearized equations are only approximately valid in the case of eccentric orbits, it would be expected that state determination accuracy would be degraded and hence more fuel and/or time needed to rendezvous.

The main components of the measurement error, namely, sextant, window and operator error, were all lumped together. Errors were introduced by rounding sextant angles σ and δ to the nearest 2, 4, or 6 mr, giving rectangular density

functions for the error, with standard deviations of .58, 1.15 and 1.73 mr respectively. With a finite number of readings per simulation and a finite number of runs, some anomalies can occur. For example, if the exact measurement was 10.2 mr, it was rounded to 10 mr at the lowest level and to 12 mr at the intermediate levels, giving errors of .2 and 1.8 mr respectively. Hence, instead of doubling the error as desired, the increase would be nine-fold in this case. The results must be interpreted with this in mind.

Thrusting may introduce control errors in burn direction and/or burn magnitude, which may be either random or biased. To simulate burn direction errors, the principal components of the desired burn were held fixed, while perpendicular components were introduced. The magnitude of the component was made equal to the product of the principal component and the burn direction error in radians, while the sign of each was chosen at random on the reference mission of a test case and kept unchanged thereafter. Of course, the astronaut is unaware of the error at the time of the burn. In the case of burn magnitude error, the direction of the desired burn remained unchanged, but the magnitude was adjusted by the product of the desired magnitude and the thrust error, expressed as a fraction. Since the intention was to simulate a fixed thruster bias, all burns of a given test case were in error by the same proportion (either all high or all low). Again the astronaut is unaware of the error and makes all subsequent calculations using his estimate of the burn magnitude rather than the true magnitude. Because the dimensionalizing scale factor on the astronaut's estimated state is proportional to his estimate of the calibrated thrust maneuver magnitude, his error in state will be directly proportional to the error in the burn. At the same time, all the burns called for in the subsequent flight plan will be in error by the same proportion. When a burn is executed, the error in incremental velocity called for will be balanced by the error in incremental velocity achieved. Thus, in theory, one expects no mission degradation whatsoever due to a fixed (but unknown) bias in thrust magnitude.

All of the perturbing influences discussed above were tested over a span which is larger than is expected in practice, simply to stress the rendezvous system heavily, and thereby determine if it breaks down under these conditions.

RESULTS

The results of the study are shown in Figs. 5-9. These figures display the ΔV and cost of each mission as a function of the magnitude of the individual error source. Since interest centers in the excess of ΔV or cost relative to the reference mission, the

results are normalized by dividing by the reference mission values. The amount by which the ΔV or cost curve exceeds unity indicates the degree to which that particular error source imposes a penalty on the mission. A curve which remains essentially at unity over the test span means that the error source does not appreciably affect the operation of the rendezvous system. Minor deviations from unity can be due to errors in the simulation itself, finite rendezvous tolerance, etc., and are of no real consequence.

The first, and most significant, result is that rendezvous was achieved in all missions of each test case -- no breakdowns occurred. Secondly, over the range of perturbing influences tested, the degradation in ΔV or cost was not overly large.

The results for case 1 show that very little degradation occurs for any of the error sources tested. Note that the ΔV curves have very nearly a unity ratio with their reference values on all five plots, Figs. 5-9. This desirable result reflects the solidity, or unperturbability, of the coelliptical approach trajectory. The circularization at a small differential altitude (see Fig. 2 at $t = 3.12$ hrs) gives the astronaut time to make a good state determination when the range is relatively short and the effect of errors is correspondingly diminished. The accumulated effect of prior errors, i.e., those made at long range, is readily removed without significant fuel or time penalty.

All three cases showed relatively little deviation in ΔV or cost in the range test (Fig. 5). Apparently the range error, although it exists, is not serious from an overall mission point of view. The bias in burn magnitude (Fig. 6) produced no appreciable effect on any of the cases, substantiating the predictions of the theory.

Case 3 was not greatly affected by any size error in burn direction (Fig. 7). Case 2 (same figure) shows some degradation, roughly proportional to the burn direction error magnitude. Similarly case 3 showed very little degradation due to measurement error (Fig. 8), while case 2 exhibits some degradation at the highest levels.

In the eccentricity test (Fig. 9), values up to and including $e = .05$ were tried. An eccentricity of .05 in this context corresponds to carrying out rendezvous with a target in an orbit whose perigee is 75 nm and whose apogee is 445 nm above the earth's surface, a rather extreme orbit for a close-earth satellite. Yet even in this severe situation, the manual rendezvous system, using the constant-coefficient linear equations derived for circular orbits, performed satisfactorily. The ΔV penalty varied from 19% for case 1 to 55% for case 2 at the highest value of eccentricity.

Trajectories from individual error runs are shown in Figs. 10-15 to illustrate more fully the details of what happens when error sources are acting. In each one of this series of figures, two trajectories are shown. These are true values of the reference trajectory and the "error" trajectory, i.e., the trajectory resulting from the introduction of a perturbing source. No plot of the astronaut's estimated trajectory is shown, but the latter can often be inferred by the reader from the data. Figure 10 shows the effects of a 1.73 mr measurement error on case 1. Errors in the state following the measurement at $t = 2.15$ result in a slightly faulty flight plan, which, instead of being coelliptical (a straight line in these coordinates), wiggles up and down by a small amount. However, since neither apogee nor perigee were caused to change sign by the burns at $t = 2.29$ and 3.07 , which set up this wiggling trajectory, no loss in ΔV -efficiency was incurred. A state-determination at 3.85 hrs provides the data for the astronaut to make a midcourse corrective maneuver, specifically, a two-burn transfer to the target started upon reaching the Hohmann line and ended one-half orbit later at $t = 4.90$. Rendezvous ΔV and time are very nearly the same as on the reference case. Figure 11 shows the effects of a 1.73° error in burn direction on case 2. It is seen that the errors in the first state determination at $t = .75$ lead to a flight plan and trajectory that fall short of the target at 2.20 hrs. The second state determination immediately following reveals this state of affairs to the astronaut, who then takes corrective action by implementing a two-burn maneuver, starting at $t = 2.30$ and ending in rendezvous .2 hrs later.

One difficulty with the use of the three-burn trajectory in rendezvous flight plans is illustrated in this example. A faulty position-velocity situation which exists close to the target, say at $t = 2.25$, results from measurements made as much as one-and-one-half orbits earlier, where the range was much greater, and the chance for error larger. Unless the astronaut enters a new three-burn flight plan, which is undesirable because it adds one full orbital period to the rendezvous time and because it is subject to the growth of errors during this interval, this positional situation must be corrected by a relatively expensive two-burn flight plan. The navigational data for the two-burn plan is sufficiently accurate, resulting from the second state-determination based on measurements made close to the target; it is the fuel used to correct the positional situation which is objectionable. There is, perhaps, some psychological penalty as well, since the astronaut is bearing down on the target at relatively high speed, and will not know his state accurately until he is almost at the point where he should have rendezvoused, namely, at $t = 2.25$. Although it takes longer, the coelliptical approach used in case 1 does not suffer from these problems.

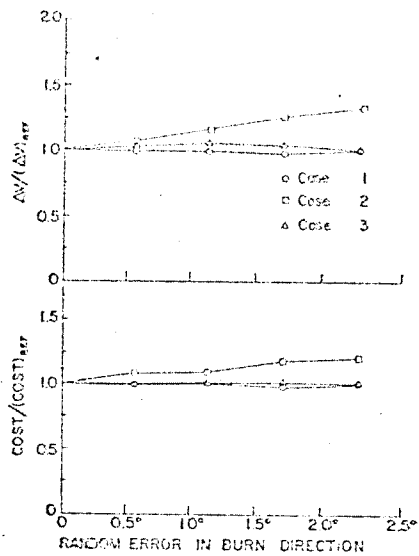


FIG. 7. MISSION PERFORMANCE RELATIVE TO ERROR IN BURN DIRECTION

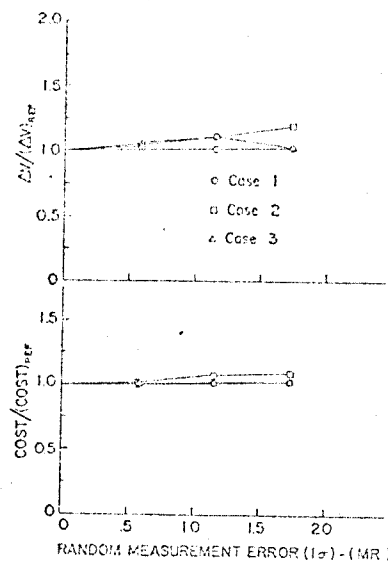


FIG. 8. MISSION PERFORMANCE RELATIVE TO MEASUREMENT ERROR

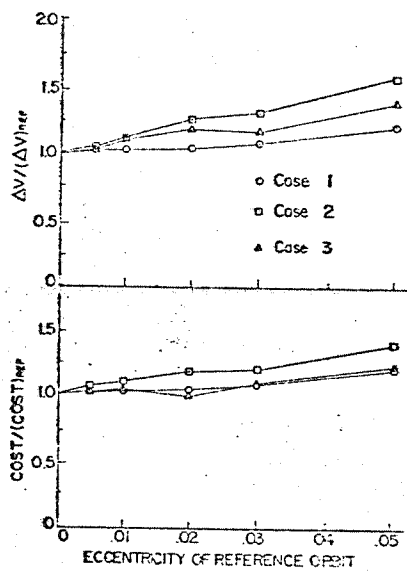


FIG. 9. MISSION PERFORMANCE RELATIVE TO REFERENCE ORBIT ECCENTRICITY

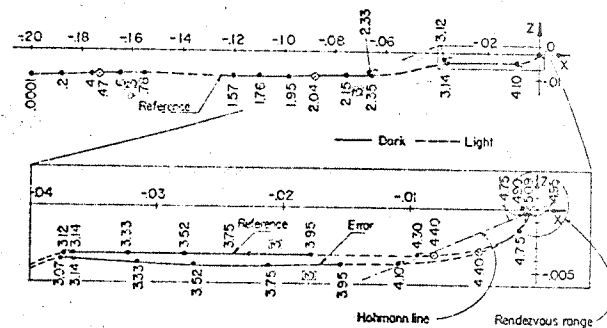


FIG. 10. ERROR TRAJECTORY - CASE 1
MEASUREMENT ERROR = 1.73 MR

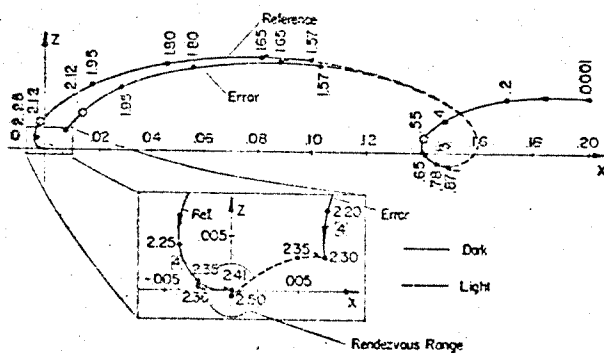


FIG. 11. ERROR TRAJECTORY - CASE 2
BURN DIRECTION ERROR = 1.73°

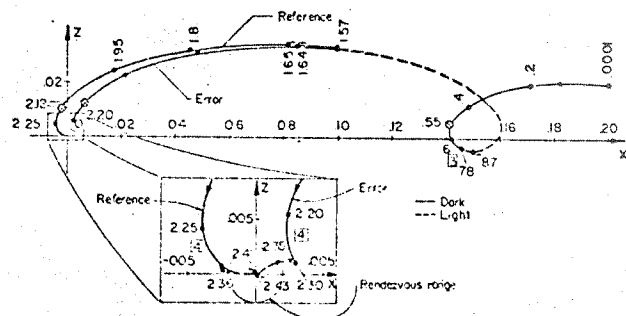


FIG. 12. ERROR TRAJECTORY - CASE 2
MEASUREMENT ERROR = 1.73 MR

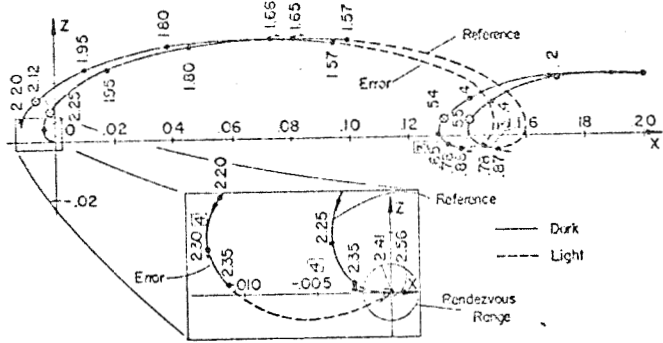


FIG. 13. ERROR TRAJECTORY - CASE 2
ECCENTRICITY = .03

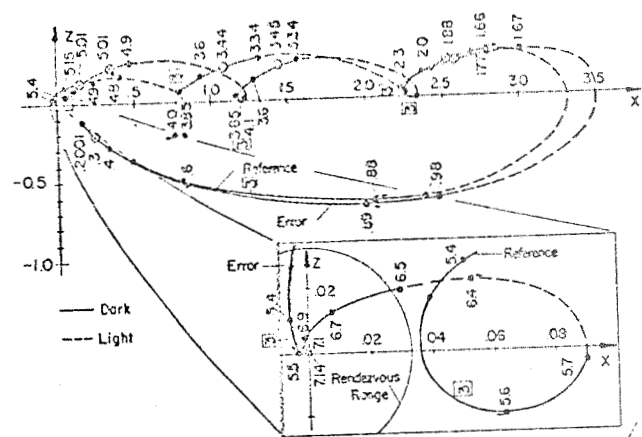


FIG. 14. ERROR TRAJECTORY - CASE 3
ECCENTRICITY = .02

56

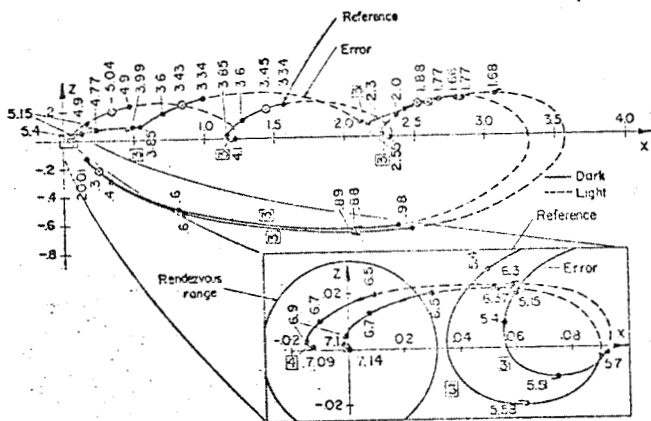


FIG. 15. ERROR TRAJECTORY - CASE 3
ECCENTRICITY = .03

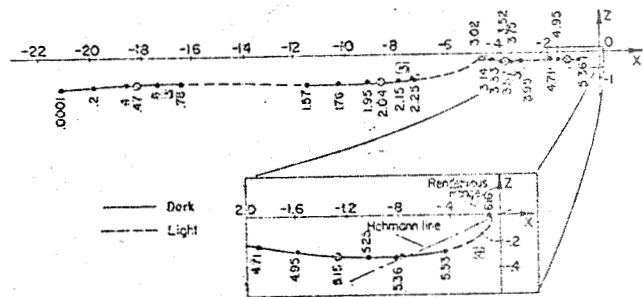


FIG. 16. ERROR TRAJECTORY - CASE 1
MULTIPLE ERROR SOURCES

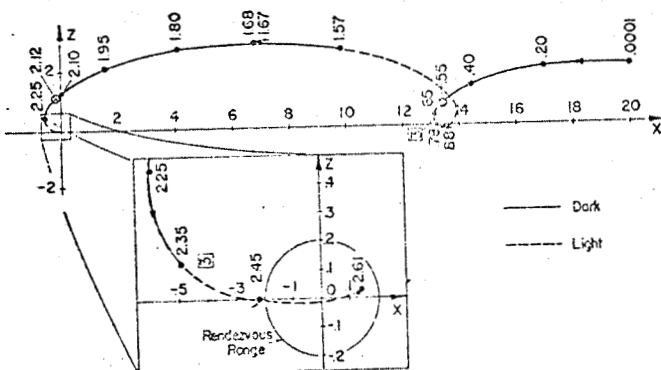


FIG. 17. ERROR TRAJECTORY - CASE 2
MULTIPLE ERROR SOURCES

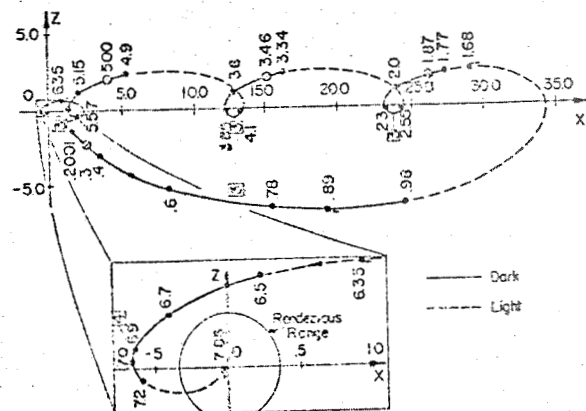


FIG. 18. ERROR TRAJECTORY - CASE 3
MULTIPLE ERROR SOURCES

Figure 12 shows the results of subjecting case 2 to a 1.73 mr measurement error. The three-burn flight plan initiated at $t = 1.87$ hrs misses slightly on the short side, and is corrected by a two-burn flight plan from $t = 2.30$ to $t = 2.43$, when rendezvous occurs.

Figure 13 shows the error trajectory for case 2 when the eccentricity is .03. The true motion in this case from $t = 0$ to $t = .88$ deviates from that of the reference case due to the inaccuracy of the dynamical equations in describing the motion⁽⁷⁾. The state-determination following the measurement at .65 hrs is followed by a three-burn flight plan, which loops around toward the target, overshooting it slightly. A midcourse state-determination following the measurement at 2.20 hrs shows the astronaut the amount he has passed the target. A two-burn plan starting at $t = 2.30$ and ending at $t = 2.56$ satisfactorily completes the rendezvous. This trajectory and the one in Fig. 11 illustrate the two possible types of erroneous approach with the looping 3-burn trajectory, overshoot and undershoot.

Figure 14 shows case 3 with an eccentricity of .02. Here the reference and error trajectories immediately begin to separate before any in-plane maneuvers are made, due to the non-exactness of the equations of motion, as discussed above. The first burn of the three-burn flight plan, made at $t = .89$, is intended to put the astronaut into a looping three-burn flight path to the target. However, a midcourse state-determination at 3.85 hrs shows that the next loop, if uncorrected, will bypass the target. Deviating from the reference strategy, a new flight plan is made, using the Flight Planning Charts. A three-burn plan is called for, with the first two burns at 4.0 and 4.8 hrs. This brings the astronaut close enough to rendezvous at $t = 5.5$ by simply reducing the relative velocity to zero, using the state-determination made at $t = 5.4$. The astronaut thus completes rendezvous one orbit earlier than on the reference mission!*

Figure 15 shows what happens to case 3 when the target orbit eccentricity is .03. Here, due to the errors in the very first state-determination, the flight plan and resulting error trajectory following the second IP burn at $t = 1.68$ had a higher apogee

* This fortuitous event, if anything, reveals that the reference case itself should have been able to rendezvous sooner. Note in Fig. 14 that at 5.5 hrs the reference trajectory very nearly came inside the rendezvous range -- another three-burn flight plan, followed by a two-burn plan, was purposely set up in the reference case to anticipate the correction mode of the typical error run.

and a higher perigee than the reference trajectory, causing the astronaut to approach the target more rapidly, in larger loops. At $t = 3.85$, a midcourse state-determination reveals to the astronaut that his originally planned trajectory will not meet the target as intended, so that (deviating from the reference strategy) a midcourse correction is necessary. After referring to the Flight Planning Charts, he starts a new three-burn plan from perigee at $t = 3.99$. However, this too fails to meet the target, because of small errors in the state-determination caused by the eccentricity. At $t = 5.56$, when he should have met the target, he is forward and slightly out of rendezvous range. Once again, based on a late state-determination at $t = 5.45$, he starts a three-burn flight plan, this time coming close enough on the third burn to complete the rendezvous at $t = 7.09$. This again shows the difficulty of the three-burn trajectory noted above -- it is hard to make a looping trajectory hit the target from a state determined by measurements made 1-1/2 or more orbits earlier.

In actual flight, all error sources act simultaneously. Three runs were made to simulate this condition, one for each of the three cases. Error sources were set to the values of Table 10, which are believed to be representative of values expected in practice. The results of these runs are shown in Table 11 and in Figs. 16-18.

TABLE 10

Values of Perturbing Inputs and Error Sources for Multi-Error Runs			
	Case 1	Case 2	Case 3
Maximum Range (nm)	21.2	20.0	34.0
Eccentricity	.015	.015	.015
Burn Bias Error (%)	4	4	4
Burn Direction Error (degrees)	1.15	1.15	1.15
Measurement Error (mr)	1.15	1.15	1.15
k (knots/rad)	0.	2.22	.594

TABLE 11

Results of Multi-Error Runs			
	Case 1	Case 2	Case 3
Total ΔV (knots)	6.3247	21.809	17.3725
Rendezvous Time (hrs)	6.16	2.61	7.85
Cost (knots)	6.3247	45.05	36.05
Reference ΔV (knots)	6.2965	21.44	17.002
Reference Rendezvous Time (hrs)	5.09	2.41	7.14
Reference Cost (knots)	6.2965	42.84	34.00
$\Delta V/(\Delta V)_{Ref.}$	1.045	1.017	1.022
Cost/(cost) _{Ref.}	1.045	1.052	1.060

The reference ΔV for a given mission was calculated by multiplying the ΔV of the corresponding reference mission by the same scale factor as that used to establish initial separation range. The reference time is the same as that on the reference mission. The value of k was determined by dividing the reference ΔV by the reference time in appropriate units. The k -value, the actual ΔV , and the actual rendezvous time were used in the standard formula (Appendix A) to compute cost.

In case 1 (Fig. 16), errors in state at 2.2 hrs result in a wiggly instead of flat trajectory from 3.02 to 5.36 hrs. The average differential altitude of this part of the trajectory is smaller than on the (scaled) reference mission, resulting in a slower approach to the target, and a later rendezvous. However, the wiggles do not significantly reduce fuel-optimality. A reasonably accurate state-determination at 3.85 hrs suffices for the (midcourse guidance) two-burn flight plan initiated at 5.36 hrs, when the astronaut's estimated state crosses the Hohmann line. The second burn of this plan at 6.16 hrs completes rendezvous in the light with a satisfactorily small miss. Actually, this miss would be accurately predicted by the state-determination made at 5.7 hrs, which would enable the astronaut to come even closer by properly planned terminal maneuvers if he chose to do so. However, since his range at this time is

.1 nm, this may not be warranted. The ΔV exceeded the reference ΔV by 4.5%; the rendezvous time exceeded the reference value by 1.07 hrs.

Case 2 results appear in Fig. 17. Following a state-determination at .75 hrs, the astronaut sets up a three-burn flight plan which should arrive at the target at $t = 2.45$. To check his progress, he redetermines state at 2.40 hrs. He predicts a small miss, which he decides to correct with a two-burn (terminal guidance) flight plan initiated at 2.45 hrs and completed in the light at 2.61 hrs at a final range of .14 nm. (In a real situation, the astronaut would see the target pass by at $t = 2.55$ at a range of about 120 feet; he would probably stop there, using visual cues.) At the rendezvous point in the figure, the ΔV exceeded the reference value by 1.7%; cost exceeded the reference value by 5.2%, reflecting the increase in time of 0.2 hrs over the reference value.

In case 3 (Fig. 18), the errors in the initial state determination at $t = .7$ lead to a looping trajectory which, instead of being tangent to the x-axis as planned, dips slightly below at $t = 2.4$ and $t = 4.0$. These deviations are recognized by the astronaut as soon as he determines state at these two times. He decides that these are small enough so that no corrective action is needed until $t = 5.57$, when, based on a state determined a few minutes earlier, he initiates a three-burn (midcourse) flight plan which is intended to bring him to the target. At $t = 6.95$ he finds that this trajectory, projected ahead to the planned rendezvous time at 7.14 hrs, misses the target, so that a new (terminal guidance) flight plan is needed. The astronaut decides that he is close enough to use a two-burn plan, which he starts at $t = 7.0$, and completes in the light at 7.85 for rendezvous at a range of approximately .02 nm (120 feet). The mission took 2.2% more ΔV than the reference case, and 6.0% more cost.

These three missions demonstrate the ability of the manual rendezvous system to function properly in the presence of realistic values of major error sources acting simultaneously. It should be noted that the effect of several simultaneous error sources is usually not as great as the sum of the effects taken one at a time -- some statistical error cancellation occurs. Also, midcourse and terminal corrections remove much of the error which has accumulated up to that point. In this respect, the manual rendezvous system acts something like a sampled-data servo, periodically nulling its errors.

STUDY OF THE MANUAL COMPUTATION PROCESS

Since the manual method is based on the astronaut

performing all computations himself, with the help of special computational aids, it is of interest to know how long these computations take, and how accurately they can be made.

A rudimentary study of the manual computation process was made, and the results, which must be regarded as preliminary, are presented. The method was to perform a complete state determination and selection of a flight plan for two different sets of initial conditions, using only the information available to the astronaut in the real situation, i. e. sextant readings, time, and knowledge of the period of one of the two vehicles. The initial conditions were selected, for convenience, to be the same as the initial conditions for cases 1 and 3, for which "sextant readings" were already available. In each case, state was determined using the applicable procedures defined in Ref. 5 (Appendix XVIII, but unlike the prior discussions, all computations were done by hand.

The person who performed this study was a senior student in the Department who had approximately ten hours of training in the system (approximately 80 hours is felt to be necessary to properly train an astronaut). It is anticipated that a fully trained user could improve in proficiency so that the computation times would be reduced below the figures given in Table 12 by 25-50%.

The following computational aids were used: a slide-rule with a ruler scale, protractor, dividers, phase-plane plotter, graph paper, Universal Trajectory Charts, ABC Tables, r - $\bar{r}$ Tables, and Flight Planning Charts.⁽⁵⁾ Manual computation times for the various steps in the process are given in Table 12, with the total time shown for one pass through the navigation and guidance equations. The steps involved would typically have to be performed more than once in any given rendezvous mission. For the mission of Case 1, an Astronaut Time-Activity Chart (Fig. 19) has been developed, closely following the events of Table 6. The time allotment for each activity was that given in Table 12, adjusted for 50% learning improvement.

An important set of calculations, the reduction of sighting angles from celestial to LTCS coordinates, was not covered in this study. The computation time would depend on which of the three possible methods suggested in Ref. 5 is implemented; the shortest time would be obtained using the mechanical spherical triangle solver⁽⁶⁾. Also not covered was the determination of windshield X-Y values for thrust alignment. In the preparation of the activity chart, 6 minutes and 1.5 minutes respectively were assumed for these calculations.

An indication of the accuracy of the manual computations is obtained by noting the differences between the hand-computed and digital computer results of Table 13. It is believed, but not verified, that his level of accuracy will not degrade mission performance to a significant degree.

TABLE 12

Manual Computations		
Designation	Case 1 Function	Time Req'd. (Min.)
IPNS	Normalized State Determination (r , $\bar{r}$ found)	10
SP ₁	Project Normalized State and In-plane Range at Two Times.	6
PC	Determination of Plane-Crossing Time.	4
SP ₂	Same as SP ₁	4
[3]	Dimensional State Determined Using 3-Instant Out-of Plane Procedure.	9
SP ₃	Projection of Dimensionalized State and Determination of In-plane Extremum Times.	5
3B	Determine Burn Magnitudes for Three-Burn Flight Plan	4
TOTAL		42
Case 3		
IPNS	Normalized State Determination (r , $\bar{r}$ found)	11
SP ₁	Project Normalized State and In-plane Range at Two Times.	6
SP ₂	Same as SP ₁	5
[3]	Dimensional State Determined Using 3-Instant Out-of-plane Procedure.	11
SP ₃	Projection of Dimensionalized State and Determination of In-plane Extremum Times.	5

TABLE 12
(Continued)

Manual Computations		
Case 3 (continued)		
3B	Determine Burn Magnitudes for Three-Burn Flight Plan	5
TOTAL		43

TABLE 13

Comparison of Hand-Calculated Values With Computer Values				
	Case 1		Case 3	
Symbol	Hand	Computer	Hand	Computer
ξ	-1.48	-1.50	-1.59	-1.62
η	-.0611	-.0530	.513	.482
r	.880	.899	.190	.168
$\bar{r}$	1.70	1.61	1.21	1.11
t_{c1}	.471	.471	1.86	1.87
t_{c2}	1.25	1.26		
t_e	.760	.782	.902	.876
PLAN	3-Burn	3-Burn	3-Burn	3-Burn

CONCLUSIONS

This study demonstrates the feasibility of the manual rendezvous system from the standpoint of being able to work successfully in the presence of realistic sources of error. The system achieves rendezvous without undue fuel or time penalty for separation ranges as large as 100 to 160 nm, for reference orbit eccentricities up to .05, for burn magnitude bias up to 8% (with theory indicating that any burn bias is acceptable), with burn direction errors up to 2.29°, and with measurement errors up to 1.73 m standard deviation.

Based on the reasonable expectation that an astronaut can be trained to perform the calculations required by the manual system in one-half the times shown in Table 12, the system is capable of being used in real time.

It is recommended that a study be made to determine methods for incorporating coelliptical flight plans for further improvement of the manual system.

APPENDIX A

SUMMARY of the MANUAL RENDEZVOUS SYSTEM*

The significant space navigation and guidance problem of rendezvous can be solved by an astronaut without the aid of sophisticated instrumentation. In the manual rendezvous method⁽¹⁾, the astronaut bases all computations on measurements taken with a hand-held sextant, a clock, and a collimated reticle pattern on the window of the spacecraft. A set of procedures has been developed which allows all data processing pertinent to the mission to be performed using charts, graphs, a slide-rule, and tables. Navigation is performed in a local tangent-to-the-flight-path coordinate system (LTCS) using linearized equations of motion to describe the relative motion of the astronaut's vehicle. The sextant measurements and a known thrust maneuver allow the astronaut to completely determine his relative position-velocity state in this frame. Guidance is performed by implementing one of three possible impulsive flight plans which minimize a linear combination of required fuel and mission time. Since errors and/or perturbations are usually present, the astronaut must continue the process by updating his state with new navigation data and adjusting his trajectory with mid-course and terminal guidance.

Linearized Dynamical Model

In order to simplify navigational computations, a model of the system dynamics was selected in which the spacecraft's equations of motion could be linearized about a nearby point in a reference orbit. The well-known linearized equations of motion for no-thrust trajectories may be written in the format:

$$\dot{X} = AX$$

whose solution for the state vector X at any non-dimensional time τ is:

$$X(\tau) = \Phi(\tau) X(0) \quad (A-1)$$

* This material is abstracted from Ref. 1

where

$$X = \begin{bmatrix} x \\ y \\ z \\ x' \\ y' \\ z' \end{bmatrix} \quad A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 0 & 0 \end{bmatrix}$$

$$\tau \equiv \omega t$$

$$t \equiv \text{time}$$

$$(\quad)' \equiv \frac{1}{\omega} \frac{d(\quad)}{dt}$$

$$\Phi(\tau) = \begin{bmatrix} 1 & 0 & 6(s-\tau) & 4s-3\tau & 0 & 2(c-1) \\ 0 & c & 0 & 0 & s & 0 \\ 0 & 0 & 4-3c & 2(1-c) & 0 & s \\ 0 & 0 & 6(c-1) & 4c-3 & 0 & -2s \\ 0 & -s & 0 & 0 & c & 0 \\ 0 & 0 & 3s & 2s & 0 & c \end{bmatrix}$$

$$s \equiv \sin \tau$$

$$c \equiv \cos \tau$$

Here x, y, z refer to the position vector components of the spacecraft relative to the reference point along the axes of a local tangent-to-the-flight path coordinate system (LTCS) in which the x -axis is forward in the reference orbital plane and tangent to the flight path, the y -axis is normal to this plane and in the direction of orbital angular velocity ω , the z -axis is mutually perpendicular to x and y in the right-handed sense, and the origin coincides with the reference point.*

Typical spacecraft trajectories resulting from the linearized model may be found in Ref. 1. All trajectories may be classified according to their shape by the parameter, r , defined by

$$r = \frac{z_a}{z_b} \quad (\text{A-2})$$

where z_a, z_b are the extreme values of z , z_a being the smaller in absolute value. Hence r ranges from -1 to $+1$. The precise location along

* This system is identical with a local vertical coordinate system when the eccentricity of the reference orbit is zero. The desirable properties of LTCS in the case of an eccentric reference orbit are discussed in Ref. 7.

the trajectory is determined by the parameter, $\bar{\tau}$, the dimensionless time interval between a particular point on a trajectory and the nearest time at which z reaches its maximum absolute value. In the absence of thrust, a vehicle will continue to traverse the same trajectory.

Measurement of Line-of-Sight Orientation

In the LTCS, the ratios of the relative position variables are dependent only on the line-of-sight orientation and not the separation range. The line-of-sight orientation in the LTCS is described by the angles σ and δ defined by

$$\cot \delta \equiv \frac{x}{z} \quad \tan \sigma \equiv \frac{y}{\sqrt{x^2 + y^2}} \quad (\text{A-3})$$

State Determination

From his measurement of σ and δ at various time instants, the astronaut would like to compute state. However, since any sequence of observed σ, δ pairs caused by some $X(t)$ could also have been caused by $KX(t)$ (in the linearized model), this computation is impossible unless a known velocity change at a known time is made, the change is observed, and the unknown K is calculated. For ease of astronaut computation, three specialized procedures have been developed for state determination. As shown in Table A-1, they differ in the sequence of measurements taken, the type of maneuver performed, and the computation algorithm used.

TABLE A-1

State Determination Methods	
Name	Method
3-instant out-of-plane burn.	Three (σ, δ) pairs and thrust in y -direction between first and third measurements.
4-instant in-plane burn.	Four (σ, δ) pairs and thrust parallel to x, z plane between either third and fourth or first and second measurements.
4-instant out-of-plane burn.	Differs from above only in that the thrust is in the y -direction at time when $y = 0$.

Flight Planning

Once the relative state of the vehicle has been determined by the navigation process, a suitable

flight plan must be developed. A flight plan specifies the number, magnitude, time and direction of the thrust impulses.

Since the out-of-plane and in-plane motion in the linearized equations are decoupled, the astronaut need only consider the in-plane rendezvous problem once the correct plane change has been effected by an out-of-plane impulse applied at any convenient crossing of the target orbital plane.

Three types of in-plane flight plans (see Table A-2) are used in the manual scheme. They were developed to satisfy a Fuel-Time Optimality Criterion while still being relatively simple to implement. Optimality is defined as minimization of a rendezvous cost equal to a linear combination of the fuel and transfer time of the mission.

$$\text{Cost} = \Delta V + k\tau.$$

Here ΔV is a measure of the expended fuel and is equal to the sum of the magnitudes of the velocity changes caused by thrust impulses. The constant k is a weighting coefficient and τ is the nondimensional transfer time (in one reference orbit period τ changes by 2π). The weighting constant k is a measure of the relative importance of fuel and transfer time. In comparing two possible flight plans, the astronaut may elect to spend up to k additional units of ΔV to save a unit of time or vice versa.

TABLE A-2

In-Plane Flight Plans	
Name	Method
Three-burn plan	Thrust impulses are directed along the local horizontal and separated in time by half-integer multiples of the reference orbit. The burns are always made at apogee or perigee of the relative trajectory.
Wait-and-two-burn plan	A minimum fuel, two-horizontal-impulse orbit transfer in which the time between impulses is unrestricted.
Immediate two-burn plan	A terminal phase, non-horizontal-impulse orbit transfer of comparatively short duration used when the fuel cost of the remaining portion of the mission is less important than the transfer time.

In use, the astronaut selects a value of k . He consults a set of Flight Planning Charts which tell him, for any state, what kind of flight plan to use.

NOMENCLATURE

A. Symbols

A	coefficient matrix of differential equation of motion
c	$\cos \tau$
Cost	combined fuel-time cost criterion
k	cost-of-time weighting constant, knots/radian
K	scaling factor for linearized model
r	shape parameter of relative trajectory
s	$\sin \tau$
t	time
T	period of reference orbit
ω	angular velocity of reference orbit, radian/hour
$\vec{\omega}$	angular velocity vector of reference orbit
x	downrange component of relative displacement, nm
x', y', z'	relative trajectory velocity components, i.e. derivative of x, y, z with respect to τ
X	6 x 1 state column vector of position and velocity, LTCS
y	out-of-plane component of relative displacement, nm
z	upward component of relative displacement, nm
z_a	the smaller in absolute value of the two extrema of z along relative trajectory
z_b	the larger in absolute value of the two extrema of z along relative trajectory
δ	in-plane angle (angle between x-axis of LTCS and projection of LOS onto plane of reference orbit)

A. Symbols (continued)

ΔV	velocity change magnitude, knots
ϵ	eccentricity
η	dimensionless vertical velocity, z'/z
ξ	dimensionless horizontal velocity, x'/x
σ	OOP angle (angle between LOS and its projection onto the reference orbit plane)
τ	normalized time (radians of travel in reference orbit), ωt
$\bar{\tau}$	dimensionless time interval (radians) between a point on a trajectory and nearest point at which $z = z_b$
Φ	state transition matrix of linearized equations of motion
[3]	3 instant out-of-plane state determination
[4]	4 instant in-plane state determination

B. Abbreviations

A_p	apogee passage
2B	two burn maneuver calculation
3B	three burn maneuver calculation
DB	dark period begins
DE	dark period ends
hrs	hours
IP	in-plane
IPNS	in-plane normalized state
LOS	line-of-sight
LTCS	local tangent-to-the-flight-path
mr	milliradians
nm	nautical miles
OOP	out-of-plane
PC	plane-crossing time
P_e	perigee passage time
rad	radians

B. Abbreviations (continued)

S^p	state projection
t_c	plane-crossing time
t_e	extremum time
<u>C. Other Notations</u>	
$(\cdot)$	differentiation with respect to t
$(\cdot)'$	differentiation with respect to τ
0	subscript, initial conditions
$\equiv$	equals by definition
$\odot$	measurement
$\otimes$	out-of-plane thrust maneuver
$\ominus$	in-plane thrust maneuver

KEY WORDS

Space
Guidance
Space Guidance
Rendezvous
Space Rendezvous Guidance
Navigation
Space Navigation
Simulation
Digital Simulation
Interactive Simulation
Digital Computer
Digital Computer, Conversational Mode
Digital Computer, Interactive
Sextant
Orbital Rendezvous
Manual Guidance

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DISCUSSION

Q. What Modelling did you use during the experiment?

A. Mathematical models were used, the modelling of the environment being contained in the digital computer. When an astronaut, who is the operator, asks for a sextant sighting for example, he would tell this to the computer and the computer would tell him what sextant sighting he would have if there had been a physical model.

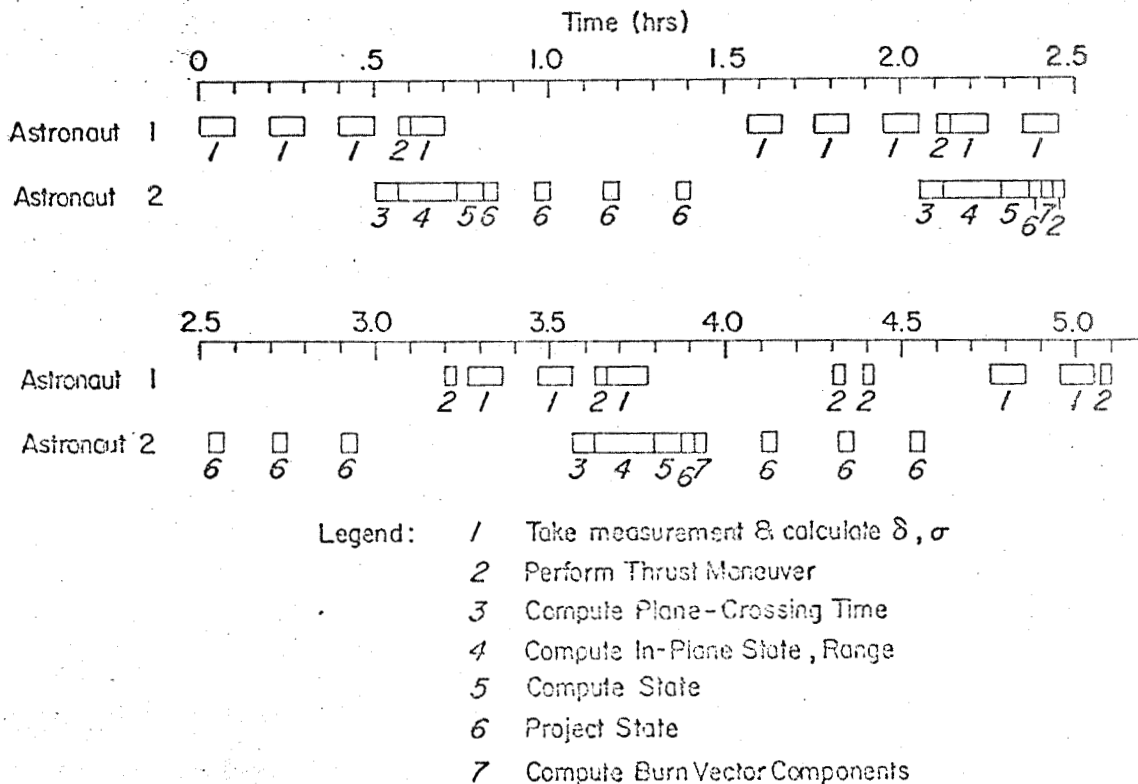


Fig. 19. ASTRONAUT TIME - ACTIVITY CHART